Critical dynamics of disordered magnets in the three-loop approximation

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(Submitted October 20, 1997; resubmitted February 16, 1998)
Fiz. Tverd. Tela (St. Petersburg) **40**, 1526–1531 (August 1998)

A field-theoretic description of the critical dynamics of magnetic systems with frozen nonmagnetic impurities is given. The values of the dynamical critical exponent in the three-loop approximation are obtained directly for three-dimensional systems using the Padé–Borel summation technique. A comparison is made with the values of the dynamical exponent for homogeneous systems calculated in the four-loop approximation as well as with the values obtained by Monte Carlo methods. © *1998 American Institute of Physics.* [S1063-7834(98)02608-2]

It is well known that phase transitions in homogeneous magnets change only for Ising magnets when randomly distributed frozen impurities are introduced into the system.[1] The ε-expansion method makes it possible to calculate the critical exponents for dilute magnets.[2] However, the asymptotic convergence of the ε-expansion series in this case is even weaker than for homogeneous magnets.[3] The renormalization-group approach to the description of disordered magnets used in Refs. 4 and 5 directly for three-dimensional systems, made it possible to obtain the values of the static critical exponents in the four-loop approximation. However, calculations to such accuracy are lacking for the description of the critical dynamics of disordered systems. This is due to the rapid growth of the volume of the calculations even in the lowest orders of perturbation theory.

In the present paper a field-theoretic description is constructed for the critical dynamics of disordered magnets directly for $d=3$ in the three-loop approximation. The model considered is a classic spin system with nonmagnetic impurity atoms frozen at the lattice sites. The model is described by the Hamiltonian

$$H=\frac{1}{2}\sum_{ij} J_{ij}p_i p_j \mathbf{S}_i\cdot\mathbf{S}_j ,$$

where $\mathbf{S}_i$ is a n-component spin variable, J_{ij} are the exchange translationally-invariant short-range ferromagnetic interaction constants, p_i is a random variable described by the distribution function

$$P(p_i)=p\,\delta(p_i-1)+(1-p)\,\delta(p_i)$$

with $p=1-c$, where c is the concentration of nonmagnetic impurity atoms. Spin-phonon interaction effects lead in the general case to nonconservation of the total spin of the system. Taking them into account, a thermodynamically equivalent Ginzburg–Landau–Wilson model determined by the following effective Hamiltonian can be introduced to describe the critical behavior of spin impurity systems

$$H[\varphi,V]=\int d^d x\left\{\frac{1}{2}[|\nabla\varphi|^2+r_0\varphi^2+V(x)\varphi^2]+\frac{g_0}{4!}\varphi^4\right\}, \tag{1}$$

where $\varphi(x,t)$ is a n-component order parameter, $V(x)$ is the potential of the random impurity field, $r_0\sim T-T_{0c}(p)$, T_{0c} is the critical temperature, determined in the mean-field theory, of a disordered magnet, g_0 is a positive constant, and d is the dimension of the system. We prescribe the impurity potential by a Gaussian distribution

$$P_V=A_V\exp\left[-(8\delta_0)^{-1}\int d^d x V^2(x)\right],$$

where A_V is a normalization constant and δ_0 is a positive constant, proportional to the impurity concentration and the squared impurity potential.

The dynamic behavior of the magnet near the critical temperature, taking account of the spin-lattice relaxation, can be described by a Langevin-type kinetic equation for the order parameter

$$\frac{\partial\varphi}{\partial t}=-\lambda_0\frac{\delta H}{\delta\varphi}+\eta+\lambda_0 h, \tag{2}$$

where λ_0 is the kinetic constant, $\eta(x,t)$ is a Gaussian random force characterizing the effect of a heat reservoir and prescribed by the distribution function

$$P_\eta=A_\eta\exp\left[-(4\lambda_0)^{-1}\int d^d x dt\,\eta^2(x,t)\right]$$

with normalization constant A_η, and $h(t)$ is an external field thermodynamically conjugate to the order parameter. The time correlation function $G(x,t)$ of the order parameter is determined by solving Eq. (2) with $H[\varphi,V]$, given by Eq. (1), for $\varphi[\eta,h,V]$ and then averaging over the Gaussian random field η by means of P_η and over the random impurity potential $V(x)$ by means of P_V and singling out the part of the solution that is linear in $h(0)$, i.e.,

$$G(x,t)=\frac{\delta}{\delta h(0)}[\langle\varphi(x,t)\rangle]_{\rm imp}\Big|_{h=0},$$

where

$$[\langle \varphi(x,t)\rangle]_{\rm imp}=B^{-1}\int D\{\eta\}D\{V\}\varphi(x,t)P_\eta P_V,$$

$$B=\int D\{\eta\}D\{V\}P_\eta P_V.$$

The application of the standard renormalization-group technique to this dynamical problem encounters substantial difficulties. However, for homogeneous systems in the absence of the disorder introduced by the presence of impurities it has been shown[6] that the critical dynamical model based on Langevin-type equations is completely equivalent to the standard Lagrangian system[7] with the Lagrangian

$$L=\int d^dx dt\left\{\lambda_0^{-1}\varphi^2+i\varphi^*\left(\lambda_0^{-1}\frac{\partial\varphi}{\partial t}+\frac{\delta H}{\delta\varphi}\right)\right\}.$$

Here the correlation function $G(x,t)$ of the order parameter for a homogeneous system is determined as

$$G(x,t)=\langle\varphi(0,0)\varphi(x,t)\rangle$$
$$=\Omega^{-1}\int D\{\varphi\}D\{\varphi^*\}\varphi(0,0)\varphi(x,t)$$
$$\times\exp(-L[\varphi,\varphi^*]),$$

where

$$\Omega=\int D\{\varphi\}D\{\varphi^*\}\exp(-L[\varphi,\varphi^*]).$$

A generalization of this field-theoretic approach and details of its application to the critical dynamics of disordered magnets with frozen point impurities and extended defects are expounded in Ref. 8. In Ref. 8 a procedure for obtaining a replica Lagrangian averaged over the impurities, the generating-functional formalism for coupled Green's functions, and the diagrammatic rules which eliminate the contribution of closed loops from the Green's functions in all orders are presented.

Instead of the correlation function it is more convenient to study its vertex part, which can be represented in the Feynman diagram formalism in the three-loop approximation in the form

$$\Gamma^{(2)}(k,\omega;r_0,g_0,\delta_0,\lambda_0)$$
$$=r_0+k^2-\frac{i\omega}{\lambda_0}-4\delta_0D_1-\frac{n+2}{18}g_0^2D_2$$
$$+\frac{4(n+2)}{3}g_0\delta_0D_3-16\delta_0^2(D_4+D_5)$$
$$+\frac{(n+2)(n+8)}{108}g_0^3\left(\sum_{i=6}^{8}D_i\right)-\frac{2(n+2)^2}{9}g_0^2\delta_0\left(\sum_{i=9}^{18}D_i\right)$$
$$+\frac{16(n+2)}{3}g_0\delta_0^2\left(\sum_{i=19}^{31}D_i\right)-64\delta_0^3\left(\sum_{i=32}^{39}D_i\right). \qquad (3)$$

The diagrams corresponding to D_i are presented in Fig. 1. The Feynman diagrams contain a d-dimensional integration over the momenta and are characterized near the critical point in the limit with the cutoff parameter $\Lambda\to\infty$ by an ultraviolet divergence at large momenta k with pole-type singularities. These poles are eliminated by using a dimensional regularization scheme, involving the introduction of renormalized quantities.[9] We shall determine the renormalized order parameter as $\varphi=Z^{-1/2}\varphi_0$. Then the renormalized vertex functions will have the generalized form

$$\Gamma_R^{(m)}(k,\omega;r,g,\delta,\lambda,\mu)=Z^{m/2}\Gamma^{(m)}(k,\omega;r_0,g_0,\delta_0,\lambda_0) \qquad (4)$$

with the renormalized coupling constants g and δ, temperature r, and kinetic coefficient λ

$$g_0=\mu^{4-d}Z_gg,\quad \delta_0=\mu^{4-d}Z_\delta\delta,$$
$$r_0=\mu^2Z_rr,\quad \lambda_0^{-1}=\mu^2Z_\lambda\lambda^{-1}. \qquad (5)$$

The scaling parameter μ is introduced to make the quantities dimensionless. In Eq. (4) $\Gamma^{(2)}$ corresponds to the reciprocal of the correlation function $G(k,\omega)$ of the order parameter, while $\Gamma^{(4)}$ corresponds to the four-tail vertex functions $\Gamma_g^{(4)}$ and $\Gamma_\delta^{(4)}$ for the coupling constants g and δ, respectively. The Z factors are determined from the requirement that the renormalized vertex functions be regular, which is expressed in the normalization conditions

$$Z\frac{\partial\Gamma^{(2)}(k)}{\partial k^2}\bigg|_{k^2=0}=1,\quad Z^2\Gamma_g^{(4)}|_{k_i=0}=\mu^{4-d}g,$$

$$Z^2\Gamma_\delta^{(4)}|_{k_i=0}=\mu^{4-d}g,\quad Z\frac{\partial\Gamma^{(2)}(k,\omega)}{\partial(-i\omega)}\bigg|_{k^2,\omega=0}=\lambda^{-1}. \qquad (6)$$

We carried out this regularization procedure for the vertex functions in the three-loop approximation. For this purpose, we present the values of the vertex functions, appearing in the normalization conditions, in the form

$$\Gamma_g^{(4)}|_{k_i=0}=g_0\sum_{i,j=0}^{3}A_{ij}g_0^i\delta_0^j,$$

$$\Gamma_\delta^{(4)}|_{k_i=0}=\delta_0\sum_{i,j=0}^{3}B_{ij}g_0^i\delta_0^j,$$

$$\frac{\partial\Gamma^{(2)}}{\partial k^2}\bigg|_{k^2=0}=\sum_{i,j=0}^{3}C_{ij}g_0^i\delta_0^j,$$

$$\frac{\partial\Gamma^{(2)}}{\partial(-i\omega/\lambda)}\bigg|_{k=0,\omega=0}=\sum_{i,j=0}^{3}D_{ij}g_0^i\delta_o^j, \qquad (7)$$

where the coefficients are sums of the corresponding diagrams or their derivatives at zero external momenta and frequencies. Thus, the numerical values of the derivatives of the diagrams (see Fig. 1) $D_i'=\partial D_i/\partial(-i\omega/\lambda)|_{k=0,\omega=0}$, generating the coefficients D_{ij} and obtained by using the subtraction scheme of Ref. 10, are presented in Table I, where $J=\int d^dq/(q^2+1)^2=(S_d/2)\Gamma(d/2)\Gamma(2-d/2)$ is a one-loop integral with $S_d=2\pi^{d/2}/(2\pi)^d\Gamma(d/2)$, and $\Gamma(x)$ is the gamma function. We write the expansion for the quantities g_0, δ_0, Z, and Z_λ in terms of the renormalized coupling constants g and δ in the form

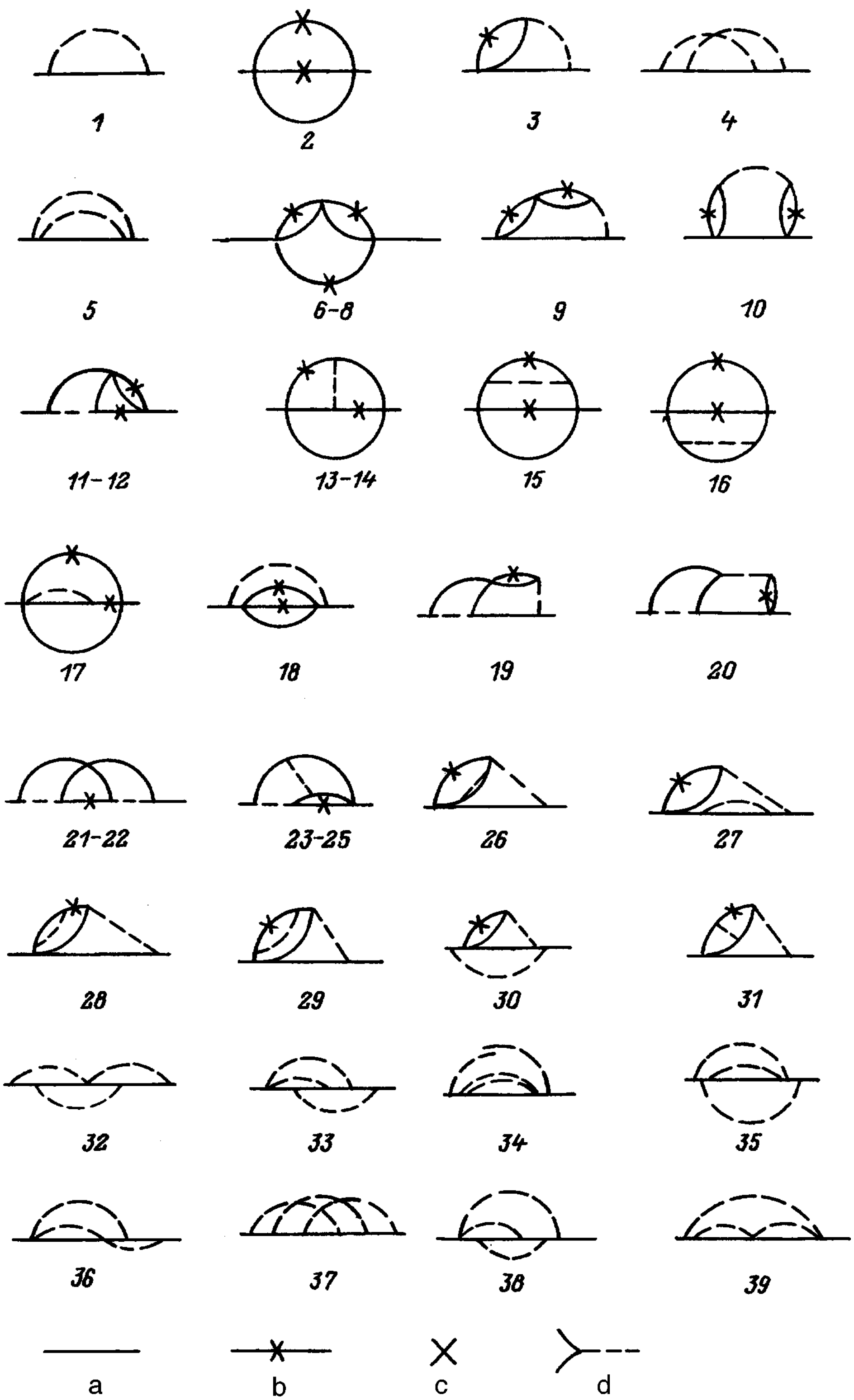

FIG. 1. Diagrammatic representation of contributions to the vertex function $\Gamma^2(k,\omega)=G^{-1}(k,\omega)$ in the three-loop approximation. The line a corresponds to $G_0(k,\omega)=(r_0+k^2-i\omega/\lambda_0)^{-1}$, the line b corresponds to $G_0(k,\omega)=2\lambda^{-1}((r_0+k^2)^2+(\omega/\lambda_0))^{-1}$, the vertex c corresponds to g_0, and the vertex d to $\delta_0\delta(\omega)$.

$$g_0=g\sum_{i,j=0}^{3} a_{ij}g^i\delta^j, \quad \delta_0=\delta\sum_{i,j=0}^{3} b_{ij}g^i\delta^j,$$

$$Z=\sum_{i,j=0}^{3} c_{ij}g^i\delta^j, \quad Z_\lambda=\sum_{i,j=0}^{3} d_{ij}g^i\delta^j, \tag{8}$$

where the unknowns a_{ij}, b_{ij}, c_{ij}, and d_{ij} are expressed in terms of A_{ij}, B_{ij}, C_{ij}, and D_{ij} by means of the normalization conditions. The next step in the field-theoretic approach is to determine the scaling functions $\beta_g(g,\delta)$, $\beta_\delta(g,\delta)$, $\gamma_r(g,\delta)$, $\gamma_\varphi(g,\delta)$, and $\gamma_\lambda(g,\delta)$, which give the renormalization-group differential equation for the vertex functions:

$$\left[\mu\frac{\partial}{\partial\mu}+\beta_g\frac{\partial}{\partial g}+\beta_\delta\frac{\partial}{\partial\delta}-\gamma_r r\frac{\partial}{\partial r}+\gamma_\lambda\lambda\frac{\partial}{\partial\lambda}-\frac{m}{2}\gamma_\varphi\right]$$
$$\times\Gamma^{(m)}(k,\omega;r,g,\delta,\lambda,\mu)=0.$$

TABLE I. Values of the derivatives of the diagrams displayed in Fig. 1, $D_i' = \partial D_i/\partial(-i\omega/\lambda)|_{k=0,\omega=0}$.

D_1'/J	−1.000000	D_{14}'/J^3	−0.032279	D_{27}'/J^3	−0.666667
D_2'/J^2	−0.130768	D_{15}'/J^3	0.061515	D_{28}'/J^3	0.584625
D_3'/J^2	−0.666667	D_{16}'/J^3	0.004666	D_{29}'/J^3	−0.092766
D_4'/J^2	−2.000000	D_{17}'/J^3	−0.333557	D_{30}'/J^3	−0.074202
D_5'/J^2	−1.000000	D_{18}'/J^3	0.042034	D_{31}'/J^3	−0.194407
D_6'/J^3	−0.104778	D_{19}'/J^3	−2.053736	D_{32}'/J^3	−2.053736
D_7'/J^3	−0.032835	D_{20}'/J^3	−2.053736	D_{33}'/J^3	−2.053736
D_8'/J^3	−0.032835	D_{21}'/J^3	−1.142275	D_{34}'/J^3	−1.000000
D_9'/J^3	−0.519431	D_{22}'/J^3	−0.396553	D_{35}'/J^3	0.666667
D_{10}'/J^3	−0.519431	D_{23}'/J^3	−1.142275	D_{36}'/J^3	0.666667
D_{11}'/J^3	−0.276601	D_{24}'/J^3	−0.396553	D_{37}'/J^3	−2.053736
D_{12}'/J^3	−0.468697	D_{25}'/J^3	−0.396553	D_{38}'/J^3	−0.074202
D_{13}'/J^3	−0.032279	D_{26}'/J^3	0.226932	D_{39}'/J^3	0.000000

In the following discussion of the dynamical behavior we shall require only the functions β_g and β_δ and the dynamical scaling function γ_γ, determined by the relations

$$4-d+\beta_g\frac{\partial \ln gZ_g}{\partial g}+\beta_\delta\frac{\partial \ln gZ_g}{\partial \delta}=0,$$

$$4-d+\beta_g\frac{\partial \ln \delta Z_\delta}{\partial g}+\beta_\delta\frac{\partial \ln \delta Z_\delta}{\partial \delta}=0,$$

$$\gamma_\lambda=\beta_g\frac{\partial \ln Z_\lambda}{\partial g}+\beta_\delta\frac{\partial \ln Z_\lambda}{\partial \delta}. \quad (9)$$

The explicit form of the functions β_g and β_δ in the four-loop approximation was obtained in Ref. 5, where the coupling constants ν and u were introduced, which are related to g and δ as $\nu=(n+8)J_g/6$ and $u=-16J\delta$. Prescribing the functions β and γ_λ in the form

$$\beta_\nu=\nu\sum_{i,j=0}^{3}\beta_{ij}^{(v)}\nu^i u^j, \quad \beta_u=u\sum_{i,j=0}^{3}\beta_{ij}^{(u)}\nu^i u^j,$$

$$\gamma_\lambda=\sum_{i,j=0}^{3}\gamma_{ij}^{i}u^j, \quad (10)$$

we present in Table II the values of the coefficients in Eq. (10) for the three-dimensional Ising model $(n=1)$. The nature of the critical point for each value of n and d is completely determined by the stable fixed point for the coupling constants (ν^*,u^*), which is determined from the requirement that the β function vanish, i.e.,

$$\beta_\nu(\nu^*,u^*)=0, \quad \beta_u(\nu^*,u^*)=0.$$

The quantities ν^* and u^* are of order $4-d$, so that the series expansions of the scaling functions with $d=3$ in powers of ν and u are asymptotically convergent. The Padé–Borel method has found wide application for summing them.[11] Numerical analysis of the equations for determining the fixed points and the conditions under which they are stable shows that in contrast to the ε-expansion with $d=3$ the accidental degeneracy of the fixed points with $n=1$ does not arise. Only two of the four fixed points are of interest: the fixed point for homogeneous systems $(\nu^*\neq 0,\ u^*\neq 0)$ and the impurity fixed point $(\nu^*\neq 0,\ u^*\neq 0)$, which determines the new critical properties of disordered magnets. The impurity fixed point is stable only for $n=1$, while for $n\geqslant 2$ the presence of disorder associated with the presence of the frozen impurities is not important for the critical behavior of magnets. The impurity fixed point point for the three-dimensional Ising model in the three-loop approximation is given by the values $\nu^*=2.256938$, $u^*=-0.728168$.

Substituting the values of the coupling constants at a fixed point into the scaling function $\gamma_\lambda(\nu,u)$ allows us to determine the dynamical critical exponent z, characterizing the critical slowing down of relaxation processes,

$$z=2+\gamma_\lambda(\nu^*,u^*). \quad (11)$$

But the series expansion of $\gamma_\lambda(\nu^*,u^*)$ in powers of ν^* and u^* with $d=3$ is, in the best case, asymptotically convergent, and to obtain reasonable values it cannot be directly summed. We summed it by the generalized Padé–Borel method, which consists of applying to the series the Borel transformation

$$\gamma_\lambda(\nu,u)=\sum_{i,j}\gamma_{ij}\nu^i u^j=\int_0^\infty e^{-t}\Gamma_\lambda(\nu t,ut)dt,$$

$$\Gamma_\lambda(x,y)=\sum_{i,j}\frac{\gamma_{ij}}{(i+j)!}x^i y^j, \quad (12)$$

and then using the Padé–Chisholm approximants

$$[M,N/K,L]=\sum_{i=0}^{M}\sum_{j=0}^{N}a_{ij}\nu^i u^j\left(\sum_{p=0}^{K}\sum_{q=0}^{L}b_{pq}\nu^p u^q\right)^{-1}.$$

TABLE II. Values of the coefficients in the expressions for the scaling functions.

(i,j)	$\beta_{i,j}^{(u)}$	$\beta_{i,j}^{(v)}$	$\gamma_{i,j}$
(0,0)	−1	1	0
(1,0)	1	3/2	−0.25
(0,1)	2/3	1	0
(2,0)	−95/216	−185/216	0.053240
(1,1)	−50/81	−104/81	0.030862
(0,2)	−92/729	−308/729	0.008400
(3,0)	0.389922	0.916667	−0.049995
(2,1)	0.857363	2.132996	−0.152964
(1,2)	0.467388	1.478058	−0.041167
(0,3)	0.090448	0.351069	−0.012642

The expansion obtained for $\gamma_\lambda(v,u)$ in powers of v and u in the three-loop approximation makes it possible to use approximants of the form $[1,1/1,1]$ and $[2,2/1,1]$. The application of the approximants $[1,1/1,1]$ corresponds to an earlier description[12] of the critical dynamics of disordered magnets in the two-loop approximation and gives the value of the dynamical exponent $z^{(2)}_{\rm imp}=2.169849$. The approximants $[2,2/1,1]$ make it possible to obtain the exponent z in the form

$$z=2+\frac{\alpha_1 u}{\beta}+\frac{\beta-1}{\beta^2}(\alpha_2 u^2+\alpha_3 uv+\alpha_4 v^2)+\frac{2\beta^2-\beta+1}{\beta^3}\times(\alpha_5 u^2 v+\alpha_6 u v^2)-\frac{1}{\beta}\left[\alpha_1 u+\frac{1}{\beta}(\alpha_2 u^2+\alpha_3 uv+\alpha_4 v^2)+\frac{1}{\beta^2}(\alpha_5 u^2 v+\alpha_6 u v^2)\right]{}_2F_0(1,1,\beta), \quad (13)$$

where ${}_2F_0(1,1,\beta)$ is the confluent hypergeometric function, while the functions α_i and β can be calculated from the following relations:

$$\alpha_1=\gamma_{1,0},\quad \alpha_2=\frac{\gamma_{2,0}}{2}-\frac{\gamma_{1,0}\gamma_{3,0}}{3\gamma_{2,0}},$$

$$\alpha_3=\frac{\gamma_{1,1}}{2}-\frac{\gamma_{1,0}\gamma_{0.3}}{3\gamma_{0,2}},\quad \alpha_4=\frac{\gamma_{0,2}}{2},$$

$$\alpha_5=\frac{\gamma_{2,1}}{6}-\frac{\gamma_{1,1}\gamma_{3,0}}{6\gamma_{2,0}}-\frac{\gamma_{2,0}\gamma_{0,3}}{6\gamma_{0,2}},$$

$$\alpha_6=\frac{\gamma_{1,2}}{6}-\frac{\gamma_{1,1}\gamma_{0,3}}{6\gamma_{0,2}}-\frac{\gamma_{0,2}\gamma_{3,0}}{6\gamma_{2,0}},$$

$$\beta=\beta_1 u+\beta_2 v,$$

$$\beta_1=-\frac{\gamma_{3,0}}{3\gamma_{2,0}},\quad \beta_2=-\frac{\gamma_{0,3}}{3\gamma_{0,2}}.$$

The use of the coupling constants at the impurity fixed point $v^*=2.256938$ and $u^*=-0.728168$ gives the following value for the dynamical exponent:

$$z^{(3)}_{\rm imp}=2.165319. \quad (14)$$

The small change in the value of the exponent $z_{\rm imp}$ calculated in the three- and two-loop approximations shows that higher-order corrections will give only very small changes. The calculations performed in Ref. 12 on the basis of the ε-expansion in the two-loop approximation gave at the same time the value $z^{(2)}_{\rm imp}=2.336$, which substantiates the need to use the renormalization-group procedure directly with $d=3$ to describe the critical behavior of dilute magnets.

In Ref. 13 we performed a calculation of the dynamical critical exponent for the homogeneous three- and two-dimensional ferromagnetic systems in the four-loop approximation on the basis of the Ginzburg–Landau–Wilson dynamical relaxation model. Specifically, for the three-dimensional Ising model the value $z^{(4)}_{\rm pure}=2.017$ was obtained for the dynamical exponent using the Padé–Borel summation technique. The large numerical differences between the values of the dynamical exponent for homogeneous and dilute Ising models make it possible to determine the effect of impurities on the dynamical critical behavior both in real physical experiments and in computer experiments using Monte Carlo methods.

Let us compare with the results of computer modeling of the dynamic critical behavior of the disordered Ising model the value obtained for the dynamical exponent $z^{(3)}_{\rm imp}$.[14–16] In Refs. 14 and 15 computer modeling of the critical relaxation of magnetization in a system with dimensions 48^3 and spin concentration $0.4\leqslant p\leqslant 1$ was performed. The Monte Carlo method combined with the dynamical renormalization-group method[17] was used to determine the dynamical critical exponent z. For homogeneous and weakly disordered systems with $p=0.95$ and 0.8, the following values were obtained for the exponents: $z(1.0)=1.97\pm0.08$, $z(0.95)=2.19\pm0.07$, and $z(0.8)=2.20\pm0.08$, which are in good agreement with our computational results. In Ref. 16 the values of the exponent z were obtained on the basis of an analysis of the asymptotic properties of the dynamical autocorrelation function for a system in a state of equilibrium and demonstrating strong fluctuations of the magnetization. Thus, the following values were obtained: $z(1.0)=2.095\pm0.008$ for a uniform system, $z(0.95)=2.16\pm0.01$, $z(0.9)=2.232\pm0.004$, and $z(0.8)=2.38\pm0.01$ for weakly disordered systems, and $z(0.6)=2.93\pm0.03$ with $p=0.6$. Adhering to the concept that the fixed point of the critical behavior of weakly disordered systems, which does not depend on the impurity concentration, is also a fixed point for any impurity concentration, in Ref. 16 the asymptotic value of the dynamical exponent was estimated to be $z=2.4\pm0.1$. The value of the exponent z obtained in Ref. 16 for a homogeneous system strongly conflicts with the results of the field-theoretic approach, while for $p=0.95$ the agreement between the values is good. Our point of view concerning the universality of the critical behavior of disordered systems was stated in Refs. 14 and 15, where we suggested that the universal critical behavior of weakly disordered systems be determined from the analogous behavior for strongly disordered systems and we advanced the hypothesis of step universality of critical exponents for three-dimensional disordered systems.

The prediction of the theory as to the effect of impurities on the dynamical critical behavior of magnets (higher value of $z_{\rm imp}(d=3)$ compared with $z_{\rm pure}(d=3)$) can be detected in a number of experimental methods: inelastic neutron scattering—the linewidth $\omega_\varphi\propto|T-T_c|^{z\nu}$ at $q=0$ and $\omega_\varphi\propto q^z$ at $T=T_c$; ESR and NMR magnetic resonance—the width of the resonance line $\Delta\omega\propto|T-T_c|^{(d-2+\eta-z)\nu}$, where η is the Fisher exponent; measuring the dynamical susceptibility for a high-frequency external magnetic field $\chi(\omega)\propto\omega^{-\gamma/z\nu}$ at $T=T_c$, where γ is the susceptibility exponent; ultrasonic experiments, where the sound absorption coefficient $\alpha(\omega)\propto|T-T_c|^{-(\alpha+z\nu)}w^2 g(\omega/|T-T_c|^{z\nu})$, the sound dispersion $C^2(\omega)-C^2(0)\propto|T-T_c|^{-\alpha}f(\omega/|T-T_c|^{z\nu})$. Unfortunately, we know of no works where an experimental investigation of the dynamical critical behavior of weakly dilute Ising-like magnets was performed.

This work was supported by the Russian Fund for Fundamental Research under Grant No. 97-02-16124.

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Translated by M. E. Alferieff