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Multicritical Behavior of Disordered Systems with Two Order Parameters

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Abstract—A field-theoretic description of phase transitions in disordered systems with two coupled order parameters is given. An analysis of renormalization-group functions is performed directly for three-dimensional systems in a two-loop approximation by using the Pade–Borel summation technique. Fixed points are found corresponding to stable multicritical behavior. The effect of frozen point impurities on the phase diagrams of the system is studied. © *2000 MAIK “Nauka/Interperiodica”.*

1. INTRODUCTION

There is a large class of systems [1, 2] in which a phase transition is observed which cannot be described with a single order parameter transforming according to an irreducible representation. Phase diagrams of such systems have a singular multicritical (bicritical or tetracritical) point. At the bicritical point, two lines of second-order phase transitions and one line of a first-order phase transition meet, whereas four lines of second-order phase transitions meet at the tetracritical point. In the immediate vicinity of a multicritical point, the system exhibits special critical behavior characterized by competition between different types of ordering. In the case of bicritical behavior, one type of ordering of the system suppresses the other, whereas in the case of tetracritical behavior, a mixed phase may occur in which both types of ordering coexist.

A model Hamiltonian of a system with two coupled order parameters ϕ and ψ transforming according to two different irreducible representations of dimensions n and m, respectively, has the form

$$\mathcal{H}_0 = \int d^d x \left(\frac{1}{2}[r_1\phi^2 + r_2\psi^2 + (\nabla\phi)^2 + (\nabla\psi)^2] + \frac{u_{10}}{4!}(\phi^2)^2 + \frac{u_{20}}{4!}(\psi^2)^2 + \frac{2u_{30}}{4!}\phi^2\psi^2 \right),$$

$$\phi^2 = \sum_{i=1}^{n}\phi_i^2, \quad \psi^2 = \sum_{i=1}^{m}\psi_i^2,$$

$$(\nabla\phi)^2 = \sum_{i=1}^{n}(\nabla\phi_i)^2, \quad (\nabla\psi)^2 = \sum_{i=1}^{m}(\nabla\psi_i)^2. \quad (1)$$

The problem of a phase transition in such a system was treated in [3, 4] by the ε-expansion method in a one-loop approximation. Recently, with the aim of refining the dependence of the multicritical behavior on the structure of order parameters, we have performed [5] a direct field-theoretic description of a three-dimensional system in terms of the Hamiltonian (1) in a two-loop approximation, without resorting to ε expansion. Studies of critical phenomena show [6] that this approach allows one to most adequately describe the critical behavior. Very accurate results can be obtained by applying this method in a many-loop approximation in combination with methods for summation of asymptotically convergent series. Using the Pade–Borel summation technique, analysis of renormalization group functions was made in [5] in a two-loop approximation and fixed points were determined which correspond to stable bicritical and tetracritical behavior. The coordinates of the fixed points and the conditions for their stability differ essentially from those found in [3, 4], which leads to noticeable changes in the phase diagrams in the critical region and to other types of symmetry of the system at the multicritical point.

In this paper, we investigate the effect of frozen point impurities on the multicritical behavior of a system with two coupled order parameters. It is known [7] that the disorder caused by frozen impurities in a system may be in the form of random fluctuations of the local critical temperature or in the form of random fields. The statistical properties of disordered systems differ essentially in these two cases, because random fields break down the symmetry of the system to the sign reversal of an order parameter. The systems with disorder of the random critical-temperature type are exemplified by ferro- and antiferromagnets with nonmagnetic impurity atoms in the absence of an external magnetic field, whereas anisotropic antiferromagnets with nonmagnetic impurity atoms in a uniform magnetic field have disorder of the random-field type [8]. In this paper, we investigate the multicritical behavior of systems with disorder of the critical-temperature type. Such behavior may be observed in disordered systems in which, as in MnAs [9], the sequence of phase transitions can be described in terms of two coupled order parameters of different nature corresponding to a structural and a ferromagnetic phase transition, or in *XY*-type antiferromagnets such as Cr_2TeO_6 and $KCuF_3$ [10], in which a multicritical point appears in the absence of an external magnetic field. In some cases, the description of the multicritical behavior of disordered binary alloys composed of magnetic atoms of two species with mixed exchange interaction may correspond to the assumption of disorder of the random critical-temperature type in a system with two coupled order parameters [11, 12].

Early investigations of the effect of disorder of the random-temperature type on the multicritical behavior of the system were performed in [11–13] by the ε-expansion method in the one-loop approximation. However, it was clearly demonstrated on the example of a homogeneous system [5] that the results obtained in the one-loop approximation are in rather poor agreement with the actual multicritical behavior. One would expect even more essential discrepancies in the case of disordered systems, as may be inferred from the results obtained for disordered systems with a single order parameter [14, 15]. For disordered Ising-type systems, an accidental degeneracy occurs in the set of renormalization group equations for interaction vertex functions in the one-loop approximation [16]. For these reasons, we cannot use this approximation when studying the effect of impurities on the critical behavior of disordered systems. In this paper, we apply a field-theoretic method directly to three-dimensional systems and use the two-loop approximation.

2. RESULTS AND DISCUSSION

The Hamiltonian of a system with two coupled order parameters, in which there are frozen impurities producing disorder of the random-temperature type, can be written in the form

$$\mathcal{H}[\phi, \psi] = \mathcal{H}_0[\phi, \psi] + \mathcal{H}_{\mathrm{imp}}[\phi, \psi], \tag{2}$$

where $\mathcal{H}_0[\phi, \psi]$ is the Hamiltonian (1) for the homogeneous system. The term $\mathcal{H}_{\mathrm{imp}}[\phi, \psi]$ describing the interaction of impurities with fluctuations of the order parameters is written as

$$\mathcal{H}_{\mathrm{imp}}[\phi, \phi] = \frac{1}{2}\int d^d x[V_1(x)\phi^2 + V_2(x)\psi^2]. \tag{3}$$

Here, $V_i(x)$ are the potentials of the random field of impurities characterized by a Gaussian distribution; in the case of point impurities, their correlators are

$$\begin{gathered}\langle\langle V_i(x)\rangle\rangle = 0, \quad \langle\langle V_1(x)V_1(x')\rangle\rangle = -u_{40}\delta(x-x'). \\ \langle\langle V_2(x)V_2(x')\rangle\rangle = -u_{50}\delta(x-x'), \\ \langle\langle V_1(x)V_2(x')\rangle\rangle = -u_{60}\delta(x-x').\end{gathered} \tag{4}$$

Using the method of replicas, we take an average over random configurations of impurities and reduce the problem of statistical description of the weakly disordered system to the problem of statistical description of a homogeneous system with the effective Hamiltonian

$$\begin{aligned}\mathcal{H}_{\mathrm{repl}}[\boldsymbol{\phi}, \boldsymbol{\psi}] &= \sum_{\alpha=1}^{k}\mathcal{H}_0[\phi_\alpha, \psi_\alpha] \\ &+ \frac{1}{2}\sum_{\alpha=1}^{k}\sum_{\beta=1}^{k}[u_{40}\phi_\alpha^2\phi_\beta^2 + u_{50}\psi_\alpha^2\psi_\beta^2 + 2u_{60}\phi_\alpha^2\psi_\beta^2],\end{aligned} \tag{5}$$

which contains k samples (“replicas”) of the initial Hamiltonian $\mathcal{H}_0$ of the homogeneous system and a number of extra terms (with impurity vertices u_{40}, u_{50}, and u_{60}) which describe the effective interaction of $(k \times n)$-component and $(k \times m)$-component order parameters through the impurity field. Thermodynamically, this statistical model is equivalent to the initial disordered model in the limit $k \longrightarrow 0$.

In the framework of the field-theoretic approach [17], the asymptotic critical behavior and the structure of phase diagrams in the fluctuation region are determined by the Callan–Symanzik renormalization group equation for the vertex parts of the irreducible Green’s functions. To find expressions for the (renormalization group) β functions in terms of the renormalized interaction vertices u_i ($i = 1,\ldots, 6$) involved in the renormalization group equation, we apply a common method based on the Feynman diagram technique and the renormalization procedure [18]. In the two-loop approximation, we obtain the following expressions for the β functions:

$$\begin{aligned}\beta_1(u) &= -u_1 + \frac{(n+8)}{6}u_1^2 + \frac{m}{6}u_3^2 + 24u_1u_4 \\ &- \frac{(41n+190)}{243}u_1^3 - \frac{2m}{27}u_3^3 - \frac{23m}{243}u_1u_3^2 \\ &- \frac{184m}{81}u_1u_3u_6 - \frac{16m}{9}u_3^2u_6 - \frac{(400n+2096)}{81}u_1^2u_4 \\ &- \frac{5920}{27}u_1u_4^2 - \frac{8m}{9}u_3^2u_4,\end{aligned}$$

$$\begin{aligned}\beta_2(u) &= -u_2 + \frac{(m+8)}{6}u_2^2 + \frac{n}{6}u_3^2 + 24u_2u_5 \\ &- \frac{(41m+190)}{243}u_2^3 - \frac{2n}{27}u_3^3 - \frac{23n}{243}u_2u_3^2 \\ &- \frac{184n}{81}u_2u_3u_6 - \frac{16n}{9}u_3^2u_6 - \frac{(400m+2096)}{81}u_2^2u_5 \\ &- \frac{5920}{27}u_2u_5^2 - \frac{8n}{9}u_3^2u_5,\end{aligned}$$

$$\beta_3(u) = -u_3 + \frac{2}{3}u_3^2 + \frac{(n+2)}{6}u_1u_3 + \frac{(m+2)}{6}u_2u_3$$

$$\begin{aligned}
&+4u_3u_4+4u_3u_5+16u_3u_6-\frac{5(n+m)+72}{486}u_3^3\\
&-\frac{23(n+2)}{486}u_1^2u_3-\frac{23(m+2)}{486}u_2^2u_3-\frac{(n+2)}{9}u_1u_3^2\\
&-\frac{(m+2)}{9}u_2u_3^2-\frac{20(n+m)+432}{81}u_3^2u_6\\
&-\frac{8(n+3)}{9}u_3^2u_4-\frac{8(m+3)}{9}u_3^2u_5-\frac{368}{27}u_3u_4^2\\
&-\frac{368}{27}u_3u_5^2-\frac{92(n+2)}{81}u_1u_3u_4-\frac{92(m+2)}{81}u_2u_3u_5\\
&-\frac{8(n+2)}{3}u_1u_3u_6-\frac{8(m+2)}{3}u_2u_3u_6-64u_3u_6^2\\
&-64u_3u_4u_6-64u_3u_5u_6,
\end{aligned}\tag{6}$$

$$\begin{aligned}
\beta_4(u) = {}&-u_4+16u_4^2+\frac{n+2}{3}u_1u_4+\frac{m}{3}u_3u_6\\
&-\frac{3040}{27}u_4^3-\frac{2m}{27}u_3^2u_6-\frac{8m}{3}u_3u_6^2-\frac{400(n+2)}{81}u_1u_4^2\\
&-\frac{23(n+2)}{243}u_1^2u_4-\frac{5m}{243}u_3^2u_4-\frac{184m}{81}u_3u_4u_6,
\end{aligned}$$

$$\begin{aligned}
\beta_5(u) = {}&-u_5+16u_5^2+\frac{m+2}{3}u_2u_5+\frac{n}{3}u_3u_6\\
&-\frac{3040}{27}u_5^3-\frac{2n}{27}u_3^2u_6-\frac{8n}{3}u_3u_6^2-\frac{400(m+2)}{81}u_2u_5^2\\
&-\frac{23(m+2)}{243}u_2^2u_5-\frac{5n}{243}u_3^2u_5-\frac{184n}{81}u_3u_5u_6,
\end{aligned}$$

$$\begin{aligned}
\beta_6(u) = {}&-u_6+8u_6^2+\frac{(n+2)}{6}u_1u_6+\frac{(m+2)}{6}u_2u_6\\
&+\frac{n}{6}u_3u_4+\frac{m}{6}u_3u_5+4u_4u_6+4u_5u_6-\frac{64}{3}u_6^3\\
&-\frac{4(n+2)}{3}u_1u_6^2-\frac{4(m+2)}{3}u_2u_6^2-\frac{23(n+2)}{486}u_1^2u_6\\
&-\frac{23(m+2)}{486}u_2^2u_6-\frac{368}{27}u_4^2u_6-\frac{368}{27}u_5^2u_6-32u_4u_6^2\\
&-32u_5u_6^2-\frac{n}{27}u_3^2u_4-\frac{4n}{9}u_3u_4^2-\frac{m}{27}u_3^2u_5\\
&-\frac{4m}{9}u_3u_5^2-\frac{5(n+m)}{486}u_3^2u_6-\frac{20(n+m)}{81}u_3u_6^2\\
&-\frac{92(n+2)}{81}u_1u_4u_6-\frac{92(m+2)}{81}u_2u_5u_6\\
&-\frac{16n}{9}u_3u_4u_6-\frac{16m}{9}u_3u_5u_6.
\end{aligned}$$

It is well known that the perturbation series are asymptotically convergent in this case and the quantities represented by the interaction vertices of fluctuations of the order parameters are too large for expressions (6) to be used immediately in the fluctuation region r_1, $r_2 \longrightarrow 0$. To extract the desired physical information from these expressions, we apply the generalized Pade–Borel method, which is used to find the sum of an asymptotically convergent series. The direct and inverse Borel transformations generalized to the six-dimensional case have the form

$$\begin{aligned}
f(u_1, \ldots, u_6) &= \sum_{i_1, \ldots, i_6} c_{i_1, \ldots, i_6} u_1^{i_1} u_2^{i_2} u_3^{i_3} u_4^{i_4} u_5^{i_5} u_6^{i_6}\\
&= \int_0^\infty e^{-t} F(u_1 t, \ldots, u_6 t)\,dt,
\end{aligned}$$

$$F(u_1, \ldots, u_6) = \sum_{i_1, \ldots, i_6} \frac{c_{i_1, \ldots, i_6}}{(i_1+\ldots+i_6)} u_1^{i_1} u_2^{i_2} u_3^{i_3} u_4^{i_4} u_5^{i_5} u_6^{i_6}. \tag{7}$$

To perform the analytic continuation of the Borel transform, we introduce a power series in an auxiliary variable λ

$$\begin{aligned}
&F(u_1, \ldots, u_6\lambda)\\
&= \sum_{k=0}^{\infty} \lambda^k \sum_{i_1, \ldots, i_6} \frac{c_{i_1, \ldots, i_6}}{k!} u_1^{i_1} u_2^{i_2} u_3^{i_3} u_4^{i_4} u_5^{i_5} u_6^{i_6} \lambda_{i_1+\ldots+i_6, k}
\end{aligned}\tag{8}$$

and take the Pade approximant $[L/M]$ at the point $\lambda = 1$. This technique was proposed and applied in [19] for describing the critical behavior of a number of systems characterized by several interaction vertices of fluctuations of order parameters. It was found in [19] that the Pade approximation in the variable λ conserves the symmetry of the system. This property is of importance in multivertex models.

To calculate the β functions in the two-loop approximation, we use approximant [2/1]. The nature of the multicritical behavior depends on the presence of a stable fixed point that is determined from the set of equations

$$\beta_i(u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*) = 0 \quad (i = 1, \ldots, 6). \tag{9}$$

This fixed point is stable if the real parts of the eigenvalues b_i of the matrix

$$B_{i,j} = \frac{\partial \beta_i(u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*)}{\partial u_j} \tag{10}$$

are positive.

The system of β functions obtained by calculating the sums representing them has a wide variety of fixed points for each value of n and m. The table shows stable fixed points for the physically most interesting values of n and m and also a number of fixed points unstable

Fixed points of the disordered system and the eigenvalues of the stability matrix

n	m	u_1^*	u_2^*	u_3^*	u_4^*	u_5^*	u_6^*	b_i ($i = 1, \ldots, 6$)
1	1	1.58892	1.58892	0	–0.03448	–0.03448	0	$0.4612 \pm 0.222i$, 0.0362, $0.4612 \pm 0.222i$, 0.0362
1	2	1.58892	0.93832	0	–0.03448	–0.00026	0	$0.4612 \pm 0.222i$, 0.0183, 0.0183, 0.6671, 0.0017
1	2	1.58892	0.93498	0	–0.03448	0	0	$0.4612 \pm 0.222i$, 0.0172, 0.0172, 0.6673, –0.0017
1	3	1.58892	0.82962	0	–0.03448	0	0	$0.4612 \pm 0.222i$, 0.0834, 0.0834, 0.1315, 0.6814
1	3	1.58892	1.28357	0	–0.03448	–0.07098	0	$0.4612 \pm 0.222i$, 0.3266, 0.3266, 5.9782, –3.1324
2	2	0.93832	0.93832	0	–0.00026	–0.00026	0	0.6671, 0.0017, 0.0017, 0.0005, 0.0005, 0.6671
2	2	0.93498	0.93498	0	0	0	0	0.6673, –0.0017, –0.0017, –0.0017, –0.0017, 0.6673
2	3	0.93832	0.82962	0	–0.00026	0	0	0.6671, 0.0017, 0.0659, 0.0659, 0.1315, 0.6814
2	3	0.93498	0.82962	0	0	0	0	0.6673, –0.0017 0.1315 0.6814, 0.0648, 0.0648
3	3	0.82962	0.82962	0	0	0	0	0.6814, 0.1315, 0.1315, 0.6814, 0.1315, 0.1315

in the two-loop approximation, which will be useful in subsequent analysis. The table also shows the eigenvalues of the stability matrix (10) for the corresponding fixed points.

Analysis of the nature of the fixed points and their stability allows the following conclusions to be made. In the presence of impurities in the system, the order parameters become decoupled in their fluctuations and only the tetracritical behavior with the general symmetry $SO(n) \oplus SO(m)$ of the system is stable. In the case of one-component order parameters ($n = m = 1$), the presence of impurities is crucial and leads to the critical behavior with indices corresponding to the indices of a disordered Ising model [14, 15]. As for the cases of $n = 1$, $m = 2$ and $n = 2$, $m = 2$, calculations predict stability of a fixed point for which the impurity vertices u_4^* and u_5^* are nonzero for both order parameters. However, we are inclined to believe that in higher approximations, the situation will reverse and that fixed point will become stable at which the order parameters are decoupled and the impurity vertices are nonzero only for one-component order parameters. This is indicated by the weak stability of fixed points of the former type and the weak instability of fixed points of the latter type. Furthermore, a similar situation is encountered in a study of the effect of impurities on the critical behavior of a system with a one order parameter in the two-loop approximation [14, 20]. In the case of $n, m \geq 3$, only the homogeneous fixed point is stable that is identical to a critical point of type 3 of a homogeneous system [5] and has a tetracritical character. Thus, when the order parameters of a system have two or more components, the presence of impurities produces no effect on their critical behavior, and the multicritical behavior has a tetracritical character. The presence of impurities in systems with two order parameters severely restricts the number of possible types of stable fixed points and, hence, the number of possible phase diagrams in comparison with homogeneous systems. Of fundamental importance is the restriction that disordered systems cannot have a phase diagram with a bicritical point. In these systems, critical fluctuations and fluctuations of the local critical temperature for interacting fields whose bare vertices satisfy the bicritical-behavior con-

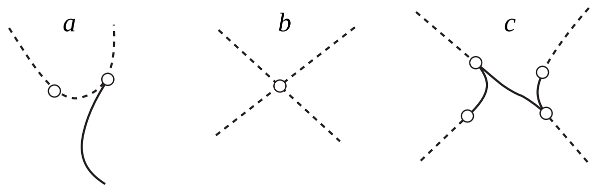

Fig. 1. Possible types of phase diagrams (schematic): solid lines correspond to first-order phase transitions and dashed lines, to second-order phase transitions.

dition $u_{30}^2 \geq u_{10}u_{20}$ [5] make the bicritical behavior unstable and result in a decoupling of the order parameters. As a consequence, phase diagrams with a bicritical character outside the critical region will contain portions of lines of first-order phase transitions in the critical region as shown in Fig. 1*a*. If the bare vertices of the system satisfy the tetracritical-behavior condition $u_{30}^2 < u_{10}u_{20}$, only phase diagrams shown in Figs. 1b and 1c are possible.

CONCLUSION

It is hoped that the differences in the multicritical behavior found here between homogeneous and disordered systems with competing order parameters will be taken into account in experimental studies of the multicritical behavior of the systems in question.

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