Multicritical behavior of weakly disordered systems with two order parameters

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(Submitted 13 February 1999)
Zh. Éksp. Teor. Fiz. **116**, 611–619 (August 1999)

We present a field-theoretic description of phase transitions in weakly disordered systems with two coupled order parameters. Using the two-loop approximation and the Padé–Borel summation technique, we carry out a renormalization-group analysis of the scaling functions for three-dimensional systems and identify the fixed points corresponding to stable multicritical behavior. We also study the effect of frozen point impurities on the nature of the phase diagrams.
 [S1063-7761(99)01808-9]

There is a broad class of systems in which the observed phase transitions cannot be described by a single order parameter transformed according to a single irreducible representation. This is especially evident in magnetic crystals where the magnetic structure is described by two or more irreducible representations (the antiferromagnets Cr_2TeO_6, $KCuF_3$, $GdAlO_3$, MnF_2, and the like). Structural phase transitions that require several order parameters for their description have been detected in $KMnF_3$, boracites, and other substances. The phase diagrams of such systems contain a singular multicritical point, which is either bicritical or tetracritical.[1,2] In the first case, two lines of second-order phase transitions intersect at this point, while in the second there are four lines of second-order phase transitions that intersect at this point. Near the multicritical point, the system exhibits specifically critical behavior characterized by competition of the different types of ordering. Here, if bicritical behavior is realized, one order parameter expels the other, while tetracritical behavior allows for a mixed phase in which both types of ordering coexist.

The model Hamiltonian of a homogeneous system with two coupled order parameters ϕ and ψ belonging to two different irreducible representations of dimensionalities n and m has the form

$$\mathscr{H}_0=\int d^dx\left\{\frac{1}{2}[r_1\phi^2+r_2\psi^2+(\nabla\phi)^2+(\nabla\psi)^2]+\frac{u_{10}}{4!}(\phi^2)^2+\frac{u_{20}}{4!}(\psi^2)^2+\frac{2u_{30}}{4!}\phi^2\psi^2\right\}, \tag{1}$$

$$\phi^2=\sum_{i=1}^{n}\phi_i^2,\quad \psi^2=\sum_{i=1}^{m}\psi_i^2,\quad (\nabla\phi)^2=\sum_{i=1}^{n}(\nabla\phi_i)^2,$$

$$(\nabla\psi)^2=\sum_{i=1}^{m}(\nabla\psi_i)^2.$$

An analysis of the problem of phase transitions in such a system was done by Lyuksyutov *et al.*[3] and (independently) by Kosterlitz *et al.*[4] in the one-loop approximation by the Wilson renormalization-group technique within the scope of the ε-expansion method, where $\varepsilon=4-d$ (d is the dimensionality of the system). Lyuksyutov *et al.*[3] attempted to follow the dependence of the multicritical behavior on the numbers n and m. However, numerous studies of systems with one order parameter done in recent years suggest that usually the one-loop approximation (and the more so when used within the ε-expansion method) yields predictions that differ substantially from real critical behavior. To establish the situation with multicritical phenomena and to find the dependence of the multicritical behavior on the structure of the order parameters, we used in Ref. 5 the two-loop approximation to provide a field-theoretic description of a system whose Hamiltonian is (1). We used the mass theory of Parisi,[6] which makes it possible to describe three-dimensional systems directly, without resorting to the ε-expansion. Studies of critical phenomena have shown[7] that such an approach provides the most meaningful description of critical behavior and its use together with methods of summation of asymptotically convergent series yields extremely accurate results. In Ref. 5 we carried out a renormalization-group analysis of the scaling functions in the two-loop approximation combined with the Padé–Borel summation technique and identified the fixed points corresponding to stable bicritical and tetracritical behavior.

For a system with two coupled order parameters there are three types of stable fixed points corresponding to different values of n and m. The corresponding domains of existence in the nm plane obtained in the one-loop approximation[3,4] are depicted in Fig. 1a and those obtained in the two-loop approximation[5] in Fig. 1b. The fact that the pattern changes so dramatically indicates that there is little resemblance between the one-loop representation and real multicritical behavior. Note that the first type corresponds to an isotropic fixed point, at which $u_1^*=u_2^*=u_3^*$ and the Hamiltonian (1) effectively coincides with the Hamiltonian of a system that has a single $(n+m)$-component order parameter and complete $SO(n+m)$ symmetry, which is higher than the $SO(n)\times SO(m)$ symmetry of the initial system (this is a manifestation of an asymptotic symmetry, due to fluctuations, at the multicritical point). For points of the second type, all three values u_i^* are finite and may not coincide. The lowest symmetry of the initial Hamiltonian, $SO(n)\times SO(m)$, corresponds to such points. Points of the third type correspond to decoupled order parameters, since at

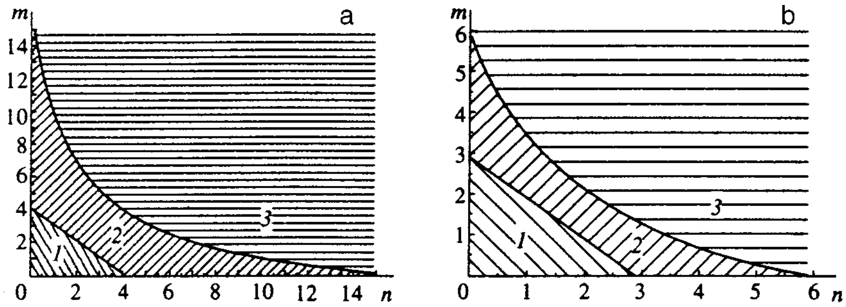

FIG. 1. The regions of stability of the fixed points determined in Refs. 3 and 4 in the first order of the ε-expansion (a) and in Ref. 5 via the field-theoretic approach in the two-loop approximation with $d=3$ (b).

these points $u_3^*=0$. Here the higher symmetry $SO(n) \oplus SO(m)$ corresponds to such points. According to Ref. 5, the edge of the stability region of an isotropic fixed point is the straight line $n+m=2.9088$, i.e., $SO(2)$ is the highest asymptotic symmetry of the system, and the domain of existence of points of the second type becomes so narrow that it contains only five physically interesting point. The appreciable change in the values of the fixed points and in the conditions for their stability, established in Ref. 5, is responsible for an appreciable change in the phase diagrams in the critical region and results in other types of the symmetry of the system at the multicritical point.

We study the effect of frozen point impurities on the multicritical behavior of a system with two coupled order parameters. As is well known,[8] the disorder in a system generated by the presence of frozen impurities manifests itself in the form of random perturbations of the local critical temperature or in the form of random fields. Because a random field breaks the symmetry of the system with respect to a change in the sign of the order parameter, the statistical properties of systems with this type of disorder differ substantially. Ferro- and antiferromagnetic systems containing nonmagnetic impurity atoms in the absence of an external magnetic field may serve as an example of disordered systems with a perturbation of the random-critical-temperature type, while in a uniform magnetic field the presence of nonmagnetic impurity atoms in anisotropic antiferromagnets manifests itself in the form of random fields.[9] In the present paper we study the multicritical behavior of system with a disorder of the random-temperature type. Such behavior can occur in disordered systems in which, as in MnAs (see Ref. 10), a sequence of phase transitions is described by introducing two coupled order parameters of different nature, parameters that correspond to the structural and ferromagnetic phase transitions, or in XY-like antiferromagnets of the type of Cr_2TeO_6, $KCuF_3$, and the like,[11] in which a multicritical point appears in a zero external magnetic field. In some cases the description of the multicritical behavior of disordered binary alloys consisting of atoms of two species with a mixed exchange interaction may correspond to the introduction of disorder of the random-critical-temperature type in a system with coupled order parameters.[12,13]

The effect of disorder of the random-temperature type on the multicritical behavior of the systems was studied by Izyumov *et al.*,[12] Laptev and Skryabin,[13] and Lisyanskiĭ and Filippov[14] by the ε-expansion method in the one-loop approximation. However, above we have clearly shown, using the example of a homogeneous system, that the predictions of the one-loop approximation do not agree with the real multicritical behavior. In disordered systems one can expect even larger discrepancies, a fact suggested by studies of disordered systems characterized by a single order parameter.[15,16] For disordered systems described by the Ising model in the one-loop approximation, accidental degeneracy occurs in the system of renormalization-group equations for the interaction vertices. This makes it impossible to use the given approximation in studies of the unique class of disordered systems in which the presence of an impurity has a real effect on the characteristics of the critical behavior of such systems. In the present paper we discuss the results of applying the field-theoretic approach in the two-loop approximation directly for three-dimensional systems.

The Hamiltonian of a system with two coupled order parameters that contains frozen impurities of the random-temperature type can be represented in the form

$$\mathscr{H}[\phi,\psi]=\mathscr{H}_0[\phi,\psi]+\mathscr{H}_{\mathrm{imp}}[\phi,\psi], \tag{2}$$

where $\mathscr{H}_0[\phi,\psi]$ is the Hamiltonian of the homogeneous system [Eq. (1)], and the term $\mathscr{H}_{\mathrm{imp}}[\phi,\psi]$, which specifies the interaction of the impurities and the order-parameter fluctuations, can be written

$$\mathscr{H}_{\mathrm{imp}}[\phi,\psi]=\frac{1}{2}\int d^dx[V_1(x)\phi^2+V_2(x)\psi^2]. \tag{3}$$

Here the $V_i(x)$ are the potentials of the random field of the impurities with a Gaussian distribution, whose correlators in the case of point impurities are given by the expressions

$$\begin{aligned}
&\langle\langle V_i(x)\rangle\rangle=0,\\
&\langle\langle V_1(x)V_1(x')\rangle\rangle=-u_{40}\delta(x-x'),\\
&\langle\langle V_2(x)V_2(x')\rangle\rangle=-u_{50}\delta(x-x'),\\
&\langle\langle V_1(x)V_2(x')\rangle\rangle=-u_{60}\delta(x-x').
\end{aligned} \tag{4}$$

Applying the replica method, we can easily average over the random distribution of impurities and reduce the problem of statistically describing a weakly disordered system to that of statistically describing a homogeneous system with the effective Hamiltonian

$$\mathscr{H}_{\mathrm{repl}}[\phi,\psi]=\sum_{\alpha=1}^{k}\mathscr{H}_0[\phi_\alpha,\psi_\alpha]+\frac{1}{2}\sum_{\alpha=1}^{k}\sum_{\beta=1}^{k}[u_{40}\phi_\alpha^2\phi_\beta^2+u_{50}\psi_\alpha^2\psi_\beta^2+2u_{60}\phi_\alpha^2\psi_\beta^2], \tag{5}$$

which contains k images (''replicas'') of the initial homogeneous component $\mathscr{H}_0$ and a number of additional terms with impurity vertices u_{40}, u_{50}, and u_{60}, which specify the effective interaction, via the impurity field, of the $(k\times n)$- and $(k\times m)$-component order parameters. This statistical model is thermodynamically equivalent to the initial disordered model in the limit $k\to 0$.

It is known that in the field-theoretic approach[17] the asymptotic critical behavior and the structure of the phase diagrams in the fluctuation region are determined by the Callan–Symanzik renormalization-group equation for the vertex parts of the irreducible Green's functions. To calculate the β-functions (the scaling functions) as functions of the renormalized interaction vertices u_i $(i=1,\dots,6)$, which enter into the renormalization-group equation, we used a standard method based on the Feynman diagrammatic technique and the renormalization procedure.[7] As a result, in the two-loop approximation, we arrived at the following expressions for the β-functions:

$$\beta_1(u)=-u_1+\frac{n+8}{6}u_1^2+\frac{m}{6}u_3^2+24u_1u_4-\frac{41n+190}{243}u_1^3$$
$$-\frac{2m}{27}u_3^3-\frac{23m}{243}u_1u_3^2-\frac{184m}{81}u_1u_3u_6$$
$$-\frac{16m}{9}u_3^2u_6-\frac{400n+2096}{81}u_1^2u_4$$
$$-\frac{5920}{27}u_1u_4^2-\frac{8m}{9}u_3^2u_4,$$

$$\beta_2(u)=-u_2+\frac{m+8}{6}u_2^2+\frac{n}{6}u_3^2+24u_2u_5$$
$$-\frac{41m+190}{243}u_2^3-\frac{2n}{27}u_3^3-\frac{23n}{243}u_2u_3^2$$
$$-\frac{184n}{81}u_2u_3u_6-\frac{16n}{9}u_3^2u_6$$
$$-\frac{400m+2096}{81}u_2^2u_5-\frac{5920}{27}u_2u_5^2-\frac{8n}{9}u_3^2u_5,$$

$$\beta_3(u)=-u_3+\frac{2}{3}u_3^2+\frac{n+2}{6}u_1u_3+\frac{m+2}{6}u_2u_3+4u_3u_4$$
$$+4u_3u_5+16u_3u_6-\frac{5(n+m)+72}{486}u_3^3$$
$$-\frac{23(n+2)}{486}u_1^2u_3-\frac{23(m+2)}{486}u_2^2u_3$$
$$-\frac{n+2}{9}u_1u_3^2-\frac{m+2}{9}u_2u_3^2$$
$$-\frac{20(n+m)+432}{81}u_3^2u_6-\frac{8(n+3)}{9}u_3^2u_4$$
$$-\frac{8(m+3)}{9}u_3^2u_5-\frac{368}{27}u_3u_4^2-\frac{368}{27}u_3u_5^2$$
$$-\frac{92(n+2)}{81}u_1u_3u_4-\frac{92(m+2)}{81}u_2u_3u_5$$
$$-\frac{8(n+2)}{3}u_1u_3u_6-\frac{8(m+2)}{3}u_2u_3u_6$$
$$-64u_3u_6^2-64u_3u_4u_6-64u_3u_5u_6, \qquad (6)$$

$$\beta_4(u)=-u_4+16u_4^2+\frac{n+2}{3}u_1u_4+\frac{m}{3}u_3u_6-\frac{3040}{27}u_4^3$$
$$-\frac{2m}{27}u_3^2u_6-\frac{8m}{3}u_3u_6^2-\frac{400(n+2)}{81}u_1u_4^2$$
$$-\frac{23(n+2)}{243}u_1^2u_4-\frac{5m}{243}u_3^2u_4-\frac{184m}{81}u_3u_4u_6,$$

$$\beta_5(u)=-u_5+16u_5^2+\frac{m+2}{3}u_2u_5+\frac{n}{3}u_3u_6-\frac{3040}{27}u_5^3$$
$$-\frac{2n}{27}u_3^2u_6-\frac{8n}{3}u_3u_6^2-\frac{400(m+2)}{81}u_2u_5^2$$
$$-\frac{23(m+2)}{243}u_2^2u_5-\frac{5n}{243}u_3^2u_5-\frac{184n}{81}u_3u_5u_6,$$

$$\beta_6=-u_6+8u_6^2+\frac{n+2}{6}u_1u_6+\frac{m+2}{6}u_2u_6+\frac{n}{6}u_3u_4$$
$$+\frac{m}{6}u_3u_5+4u_4u_6+4u_5u_6-\frac{64}{3}u_6^3-\frac{4(n+2)}{3}u_1u_6^2$$
$$-\frac{4(m+2)}{3}u_2u_6^2-\frac{23(n+2)}{486}u_1^2u_6$$
$$-\frac{23(m+2)}{486}u_2^2u_6-\frac{368}{27}u_4^2u_6-\frac{368}{27}u_5^2u_6-32u_4u_6^2$$
$$-32u_5u_6^2-\frac{n}{27}u_3^2u_4-\frac{4n}{9}u_3u_4^2-\frac{m}{27}u_3^2u_5$$
$$-\frac{4m}{9}u_3u_5^2-\frac{5(n+m)}{486}u_3^2u_6-\frac{20(n+m)}{81}u_3u_6^2$$
$$-\frac{92(n+2)}{81}u_1u_4u_6-\frac{92(m+2)}{81}u_2u_5u_6$$
$$-\frac{16n}{9}u_3u_4u_6-\frac{16m}{9}u_3u_5u_6.$$

Perturbation-theory series are known to be asymptotically convergent and the vertices of the interaction of the order-parameter fluctuations in the fluctuation region $r_1,r_2\to 0$, are large enough so that Eqs. (6) can be used directly. Thus, to extract the necessary physical information from these expressions we employed the generalized Padé–Borel method used for the summation of asymptotically convergent series. Here the direct and inverse Borel transformations generalized to the six-parameter case and retaining the symmetry of the system have the form

TABLE I. Values of the fixed points of a disordered system and the eigenvalues of the stability matrix.

n	m	u_1^*	u_2^*	u_3^*	u_4^*	u_5^*	u_6^*	b_i ($i=1,\ldots,6$)
1	1	1.588 92	1.588 92	0	−0.034 48	−0.034 48	0	$0.4612\pm0.222i$, 0.0362, $0.4612\pm0.222i$, 0.0362
1	2	1.588 92	0.938 32	0	−0.034 48	−0.000 26	0	$0.4612\pm0.222i$, 0.0183, 0.0183,0.6671,0.0017
1	2	1.588 92	0.934 98	0	−0.034 48	0	0	$0.4612\pm0.222i$, 0.0172, 0.0172,0.6673,−0.0017
1	3	1.588 92	0.829 62	0	−0.034 48	0	0	$0.4612\pm0.222i$, 0.0834, 0.0834,0.1315,0.6814
1	3	1.588 92	1.283 57	0	−0.034 48	−0.070 98	0	$0.4612\pm0.222i$, 0.3266, 0.3266,5.9782,−3.1324
2	2	0.938 32	0.938 32	0	−0.000 26	−0.000 26	0	0.6671,0.0017,0.0017, 0.0005,0.0005,0.6671
2	2	0.934 98	0.934 98	0	0	0	0	0.6673,−0.0017,−0.0017, −0.0017,−0.0017,0.6673
2	3	0.938 32	0.829 62	0	−0.000 26	0	0	0.6671,0.0017,0.0659, 0.0659,0.1315,0.6814
2	3	0.934 98	0.829 62	0	0	0	0	0.6673,−0.0017,0.1315, 0.6814,0.0648,0.0648
3	3	0.829 62	0.829 62	0	0	0	0	0.6814,0.1315,0.1315, 0.6814,0.1315,0.1315

$$f(u_1,\ldots,u_6)=\sum_{i_1,\ldots,i_6} c_{i_1,\ldots,i_6}u_1^{i_1}u_2^{i_2}u_3^{i_3}u_4^{i_4}u_5^{i_5}u_6^{i_6}$$

$$=\int_0^\infty e^{-t}F(u_1t,\ldots,u_6t)dt, \tag{7}$$

$$F(u_1,\ldots,u_6)=\sum_{i_1,\ldots,i_6}\frac{c_{i_1,\ldots,i_6}}{(i_1+\cdots+i_6)!}u_1^{i_1}u_2^{i_2}u_3^{i_3}u_4^{i_4}u_5^{i_5}u_6^{i_6}.$$

To analytically continue the Borel image of a function, we introduce a power series in an auxiliary parameter λ,

$$\tilde{F}(u_1,\ldots,u_6,\lambda)=\sum_{k=0}^{\infty}\lambda^k\sum_{i_1,\ldots,i_6}\frac{c_{i_1,\ldots,i_6}}{k!}\times u_1^{i_1}u_2^{i_2}u_3^{i_3}u_4^{i_4}u_5^{i_5}u_6^{i_6}\delta_{i_1+\cdots+i_6,k}, \tag{8}$$

to which we apply the Padé [L/M] approximation at the point $\lambda=1$. To calculate the β-functions in the two-loop approximation, we use the [2/1] approximant. Multicritical behavior is determined by the existence of a stable fixed point satisfying the system of equations

$$\beta_i(u_1^*,u_2^*,u_3^*,u_4^*,u_5^*,u_6^*)=0,\quad i=1,\ldots,6. \tag{9}$$

The requirement that a fixed point be stable reduces to the requirement that the eigenvalues b_i of the matrix

$$B_{i,j}=\frac{\partial\beta_i(u_1^*,u_2^*,u_3^*,u_4^*,u_5^*,u_6^*)}{\partial u_j} \tag{10}$$

lie in the right complex half-plane.

The resulting system of summed β-functions for each n and m contains a broad spectrum of fixed points. Table I lists stable fixed points of a system for the values of n and m of the uppermost interest to physics and a number of fixed points that are unstable in the two-loop approximation, which we will need in our further analysis. Table I also lists the eigenvalues of the stability matrix for the corresponding fixed points.

An analysis of the nature of the fixed points and their stability suggests the following: the presence of impurities in the system causes fluctuation decoupling of the order parameters and ensures only one type of stable multicritical behavior, the tetracritical behavior with the common symmetry of the system being $SO(n)\oplus SO(m)$. Here, for one-component order parameters ($n=m=1$), the presence of impurities is important and leads to a critical behavior with exponents corresponding to those of the disordered Ising model.[16] As for the cases with $n=1$, $m=2$, and $n=2$, $m=2$, although calculations show that the fixed point with finite values of the impurity vertices u_4^* and u_5^* for both order parameters is stable, we believe that in the higher orders of the approximation the fixed point that is stable is the one at which, for the same general effect of decoupling of the order parameters, the values of the impurity vertices are finite only for one-component order parameters. A possible indication of this is, on the hand, the weak stability of fixed points of the first type accompanied by the weak instability of fixed points of the second type and, on the other, that a similar situation arises in the analysis of the effect of impurities on the critical behavior of systems with one order parameter in the two-loop approximation.[18] For $n,m\geqslant 3$ only the homogeneous fixed point that is stable coincides with the third-type point of a homogeneous system is tetracritical in nature. Thus, when the order parameters of the system are characterized by a number of components that is larger than, or equal to, two, the presence of impurities has no effect on the characteristics of their critical behavior and the multicritical behavior is tetracritical.

The phase diagrams for the Hamiltonian (1) of a homogeneous system in the mean-field approximation (i.e., without fluctuations) are well known (see, e.g., Ref. 3). For in-

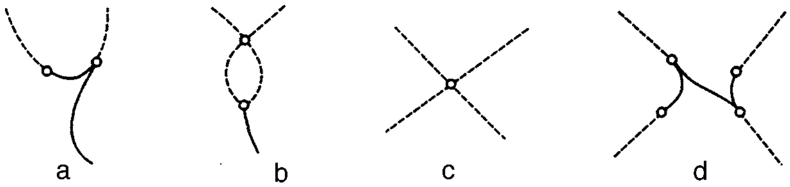

FIG. 2. Possible phase-diagram types. The solid lines correspond to curves representing first-order phase transitions, while the dashed lines correspond to curves representing second-order phase transitions.

stance, when $u_3^2 < u_1 u_2$ holds the tetracritical point is realized, with the result that there cam be a mixed phase with $\phi \neq 0$ and $\psi \neq 0$. In the opposite case, $u_3^2 \geqslant u_1 u_2$, the phase diagram has a bicritical point and there is no mixed phase. However, as shown in Refs. 3 and 5, allowance for fluctuations may dramatically change the phase diagram in the critical region. To establish whether this is the case, we must study the phase portrait of the system on the basis of the solution of the system of equations $(r = r_1 = r_2)$

$$r \frac{\partial u_i}{\partial r} = \beta_i(u_j), \tag{11}$$

which specifies the phase trajectories in the space of the vertices u_i. As $r \to 0$, depending on the bare values of the vertices, u_{i0}, either the phase trajectories leave the stability region of the Hamiltonian (1) and a first-order phase transition takes place, or they end up at a stable fixed point from the set of the points considered above with a definite symmetry of the system. In their motion the phase trajectories may cross regions in which the vertices meet the condition for tetra- or bicriticality. As a result, in the critical region on the phase diagrams corresponding to the mean-field theory there appear segments of curves representing first-order phase transitions.

Determining the regions for the values of the vertices u_i of the replica Hamiltonian (5) of a disordered systems in which the system is stable is complicated by the need to examine the limit $k \to 0$. However, allowing for the fact that the impurity vertices u_{40}, u_{50}, and u_{60} are proportional to $c(1-c)$, where c is the concentration of the impurity atoms, and limiting ourselves to weakly disordered systems, we assume that the former stability conditions ($u_{10} > 0$, $u_{20} > 0$, and $u_{30} > -(u_{10} u_{20})^{1/2}$) and the conditions for tetracritical or bicritical behavior are met, just as they are for a homogeneous system. The only additional condition imposed on the impurity vertices u_{40}, u_{50}, and u_{60} is that they be negative, a requirement that follows from the fact that the corresponding correlators in (4) are positive. In view of this, when we construct the phase portrait and the possible phase diagrams for weakly disordered systems and allow for fluctuations in the multicritical region in the solution of Eqs. (11) with the β-functions of (6) summed by the Padé–Borel method, it is sufficient to follow the changes in the values of u_1, u_2, and u_3 and assume that they depend on the vertices u_4, u_5, and u_6 parametrically.

Because the presence of impurities in systems with two order parameters substantially limits the possible types of stable fixed points, the number of possible types of phase diagrams changes dramatically in relation to that number in homogeneous systems. What is important in such changes is that for disordered systems there can be no phase diagram with a bicritical point. For disordered systems with interacting fields whose vertices have bare values that meet the bicriticality condition $u_{30}^2 \geqslant u_{10} u_{20}$, critical fluctuations and fluctuations of the local critical temperature destroy the stability of bicritical behavior and decouple the order parameters. As a result, the phase diagrams with bicritical behavior outside the critical region contain segments of curves representing first-order phase transitions, with the corresponding diagram being the one in Fig. 2a. The numerical solution of Eq. (11) shows that for all bare values of the vertices, u_{i0}, lying in the bicritical region there can be no phase trajectories that take these segments to a stable fixed tetracritical point. As a result, the phase diagram predicted by Laptev and Skryabin[13] and depicted in Fig. 2b is not realized. But if the bare values of the system's vertices satisfy the tetracriticality condition, the only possible diagrams are those depicted in Figs. 2c and 2d. Thus, of all the phase diagrams discussed in Refs. 3 and 5 and realized in homogeneous systems with two order parameters, only three types of phase diagram, those depicted in Figs. 2a, c, and d, are realized in disordered systems.

In conclusion we would like to express the hope that the differences in the multicritical behavior of homogeneous and disordered systems with competing order parameters established here will find their reflection in the design and analysis of experiments on the multicritical behavior of the corresponding systems.

This study was made possible by a Grant from the Russian Fund for Fundamental Research (Grant No. 97-02-16124).

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Translated by Eugene Yankovsky