

### Critical dynamics of slightly disordered spin systems

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This paper is a field theoretic description of the critical dynamics of spin systems with frozen nonmagnetic impurities. For three-dimensional systems the dynamical critical exponent is found directly by employing the three-loop approximation with the Padé–Borel summation technique. The results are compared with those obtained by calculating the dynamical exponent for homogeneous systems in the four-loop approximation, and with the values obtained by computer simulation of the critical dynamics by Monte Carlo methods. Calculations of the dynamical exponent for the two-dimensional Ising model in the four-loop approximation are also presented. © 1998 American Institute of Physics. [S1063-7761(98)01509-1]

As is known, only for Ising magnetic materials do phase transitions in homogeneous spin systems change when randomly distributed frozen nonmagnetic impurities are introduced into such systems.<sup>1</sup> The  $\epsilon$ -expansion method makes it possible to calculate the values of the critical exponents for dilute magnetic materials.<sup>2</sup> However, the asymptotic convergence of the  $\epsilon$ -expansion series in this case is even slower than for homogeneous systems.<sup>3</sup> The renormalization group approach to the description of slightly disordered spin systems, applied directly to three-dimensional systems by Mayer *et al.*,<sup>4,5</sup> has made it possible to obtain static critical exponents in the four-loop approximation. However, no calculations of equal accuracy exist for the description of the dynamics of disordered systems, the reason being that the computational load is extremely large even in the lowest perturbation-theory orders.

The present paper is a field theoretic description of the critical dynamics of slightly disordered three-dimensional spin systems in the three-loop approximation. The adopted model is a classical spin system with the nonmagnetic impurity atoms frozen at the lattice sites. The system Hamiltonian is

$$H = \frac{1}{2} \sum_{ij} J_{ij} p_i p_j \mathbf{S}_i \cdot \mathbf{S}_j,$$

where  $\mathbf{S}_i$  is an  $n$ -component spin variable,  $J_{ij}$  are the coupling constants of the translation-invariant short-range ferromagnetic interaction, and  $p_i$  is a random variable described by the distribution function

$$P(p_i) = p \delta(p_i - 1) + (1 - p) \delta(p_i)$$

with  $p = 1 - c$  (here  $c$  is the concentration of the nonmagnetic impurity atoms). Thermodynamically this model is equivalent to the  $O(n)$ -symmetric Ginzburg–Landau–Wilson model, which has the Hamiltonian

$$H[\varphi, V] = \int d^d x \left\{ \frac{1}{2} [|\nabla \varphi|^2 + r_0 \varphi^2 + V(x) \varphi^2] + \frac{g_0}{4!} \varphi^4 \right\}, \quad (1)$$

where  $\varphi(x, t)$  is the  $n$ -component order parameter,  $V(x)$  is the potential of the random impurity field,  $r_0 \sim T - T_{0c}(p)$ , with  $T_{0c}$  the system's critical temperature determined in the mean-field theory,  $g_0$  is a positive constant, and  $d$  is the number of dimensions of the system. We assume that the impurity potential is specified by a Gaussian distribution:

$$P_V = A_V \exp \left[ - (8 \delta_0)^{-1} \int d^d x V^2(x) \right],$$

where  $A_V$  is a normalization constant, and  $\delta_0$  is a positive constant proportional to the impurity concentration and the square of the impurity potential.

The dynamical behavior of the system in the relaxation regime near the critical temperature can be described by the Langevin transport equation for the order parameter:<sup>6</sup>

$$\frac{\partial \varphi}{\partial t} = -\lambda_0 \frac{\delta H}{\delta \varphi} + \boldsymbol{\eta} + \lambda_0 \mathbf{h}, \quad (2)$$

where  $\lambda_0$  is the transport coefficient,  $\boldsymbol{\eta}(x, t)$  is the Gaussian random force, which is a measure of the effect of the heat reservoir and is specified by the distribution function

$$P_\eta = A_\eta \exp \left[ - (4 \lambda_0)^{-1} \int d^d x dt \boldsymbol{\eta}^2(x, t) \right]$$

with normalization constant  $A_\eta$ , and  $\mathbf{h}(t)$  is an external field thermodynamically conjugate to the order parameter. The temporal correlation function  $G(x, t)$  of the order parameter can be found by solving Eq. (2) for  $\varphi[\boldsymbol{\eta}, \mathbf{h}, V]$ , with  $H[\varphi, V]$  given by (1), averaging the result over the Gaussian random force  $\boldsymbol{\eta}$  via  $P_\eta$  and over the random potential  $V(x)$  of the impurity field via  $P_V$ , and isolating the part of the solution that is linear in  $\mathbf{h}(0)$ , i.e.,

$$G(x, t) = \frac{\delta}{\delta \mathbf{h}(0)} \cdot \langle \varphi(x, t) \rangle_{\text{imp}}|_{\mathbf{h}=0},$$

where

$$\langle \varphi(x, t) \rangle_{\text{imp}} = B^{-1} \int D\{\boldsymbol{\eta}\} D\{V\} \varphi(x, t) P_\eta P_V,$$

$$B = \int D\{\boldsymbol{\eta}\} D\{V\} P_{\eta} P_V.$$

Significant difficulties are encountered when the standard renormalization group method is applied to this dynamical model. However, as shown by De Dominicis<sup>7</sup> for inhomogeneous systems in the absence of disorder introduced by impurities, in describing the critical dynamics the model based on the Langevin equation is equivalent to the standard Lagrange system<sup>8</sup> with the Lagrangian

$$L = \int d^d x dt \left\{ \lambda_0^{-1} \dot{\boldsymbol{\varphi}}^2 + i \boldsymbol{\varphi}^* \cdot \left( \lambda_0^{-1} \frac{\partial \boldsymbol{\varphi}}{\partial t} + \frac{\delta H}{\delta \boldsymbol{\varphi}} \right) \right\},$$

where we have introduced the auxiliary field  $\boldsymbol{\varphi}^*$ . Here the correlation function  $G(x, t)$  of the order parameter for a homogeneous system is given by

$$\begin{aligned} G(x, t) &= \langle \boldsymbol{\varphi}(0, 0) \cdot \boldsymbol{\varphi}(x, t) \rangle \\ &= \Omega^{-1} \int D\{\boldsymbol{\varphi}\} D\{\boldsymbol{\varphi}^*\} \boldsymbol{\varphi}(0, 0) \cdot \boldsymbol{\varphi}(x, t) \exp(-L[\boldsymbol{\varphi}, \boldsymbol{\varphi}^*]), \end{aligned}$$

where

$$\Omega = \int D\{\boldsymbol{\varphi}\} D\{\boldsymbol{\varphi}^*\} \exp(-L[\boldsymbol{\varphi}, \boldsymbol{\varphi}^*]).$$

A generalization of the given field theoretic approach and the details of applying such an approach to the critical dynamics of disordered spin systems with frozen point impurities and extended defects are discussed in the context of the  $\varepsilon$ -expansion in a paper by one of the present authors.<sup>9</sup>

Instead of examining the correlation function proper, it is convenient to study its vertex part, which in the context of the Feynman diagram approach can be written in the three-loop approximation as follows:

$$\begin{aligned} \Gamma^{(2)}(k, \omega; r_0, g_0, \delta_0, \lambda_0) &= r_0 + k^2 - \frac{i\omega}{\lambda_0} - 4\delta_0 D_1 \\ &- \frac{n+2}{18} g_0^2 D_2 + \frac{4(n+2)}{3} g_0 \delta_0 D_3 - 16\delta_0^2 (D_4 + D_5) \\ &+ \frac{(n+2)(n+8)}{108} g_0^3 \left( \sum_{i=6}^8 D_i \right) - \frac{2(n+2)^2}{9} \\ &\times g_0^2 \delta_0 \left( \sum_{i=9}^{18} D_i \right) + \frac{16(n+2)}{3} \\ &\times g_0 \delta_0^2 \left( \sum_{i=19}^{31} D_i \right) - 64\delta_0^3 \left( \sum_{i=32}^{39} D_i \right). \end{aligned} \quad (3)$$

The diagrams corresponding to the  $D_i$  are depicted in Fig. 1. The Feynman diagrams contain  $d$ -dimensional integrals with respect to momenta and are characterized near the critical point by an ultraviolet divergence at high momenta  $\mathbf{k}$  with pole singularities. To remove these poles one usually employs a dimensional regularization scheme, which involves introducing renormalized quantities.<sup>10</sup> We define the renormalized order parameter as  $\boldsymbol{\varphi} = Z^{-1/2} \boldsymbol{\varphi}_0$ . Then the renormalized vertex functions have the generalized form

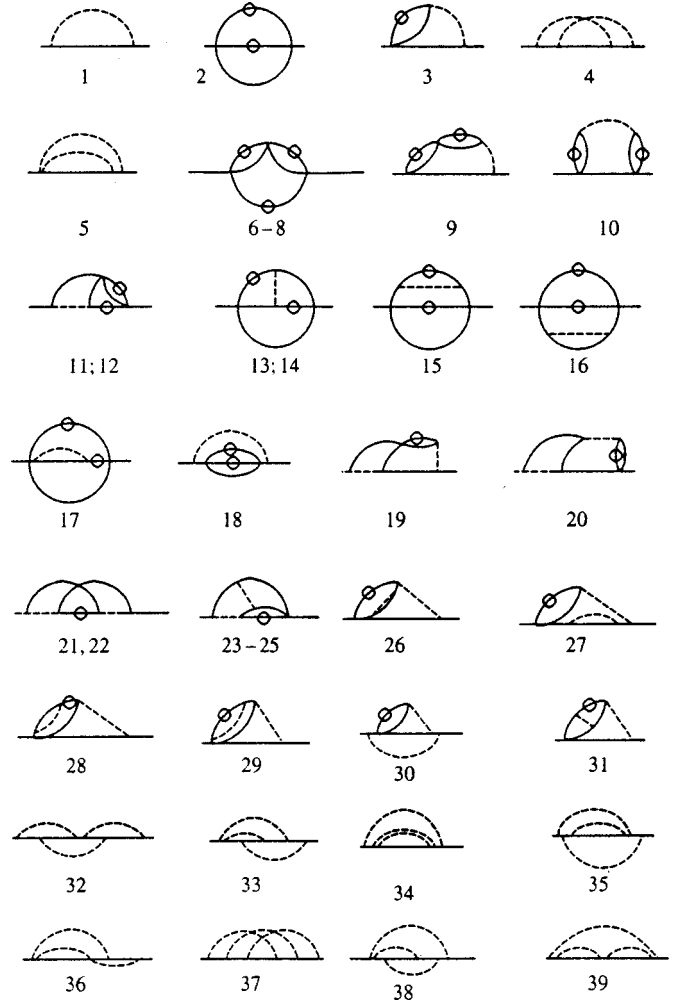


FIG. 1. Diagrammatic representation of the contributions to the vertex function  $\Gamma^{(2)}(k, \omega) = G^{-1}(k, \omega)$  in the three-loop approximation. The solid lines correspond to  $G_0(k, \omega) = (r_0 + k^2 - i\omega/\lambda_0)^{-1}$ , the solid lines with a circle correspond to  $C_0(k, \omega) = 2\lambda^{-1}((r_0 + k^2)^2 + (\omega/\lambda_0)^2)^{-1}$ , a four-leg vertex  $\times$  corresponds to  $g_0$ , and an impurity four-leg vertex  $\circ$  corresponds to  $\delta_0 \delta(\omega)$ .

$$\Gamma_R^{(m)}(k, \omega; r, g, \delta, \lambda, \mu) = Z^{m/2} \Gamma^{(m)}(k, \omega; r_0, g_0, \delta_0, \lambda_0) \quad (4)$$

with the renormalized coupling constants  $g$  and  $\delta$ , temperature  $r$ , and transport coefficient  $\lambda$ :

$$\begin{aligned} g_0 &= \mu^{4-d} Z_g g, & \delta_0 &= \mu^{4-d} Z_\delta \delta, \\ r_0 &= \mu^2 Z_r r, & \lambda_0^{-1} &= \mu^2 Z_\lambda \lambda^{-1}, \end{aligned} \quad (5)$$

where the scaling parameter  $\mu$  is introduced so that the quantities reduce to dimensionless form. In Eq. (4),  $\Gamma^{(2)}$  corresponds to the reciprocal correlation function of the order parameter  $G(k, \omega)$ , and  $\Gamma^{(4)}$  corresponds to the four-leg vertex functions  $\Gamma_g^{(4)}$  and  $\Gamma_\delta^{(4)}$  for the coupling constants  $g$  and  $\delta$ , respectively; the  $Z$ -factors can be found from the requirement that the renormalized vertex functions be regular, which is expressed in the normalization conditions

$$Z \frac{\partial \Gamma^{(2)}(k)}{\partial k^2} \Big|_{k^2=0} = 1, \quad Z^2 \Gamma_g^{(4)} \Big|_{k_i=0} = \mu^{4-d} g,$$

TABLE I. Values of the derivatives of the diagrams depicted in Fig. 1,  $D'_i = \left. \frac{\partial D_i}{\partial(-i\omega/\lambda)} \right|_{k=0, \omega=0}$ .

$D'_1/J$	-1.000 000	$D'_{14}/J^3$	-0.032 279	$D'_{27}/J^3$	-0.666 667
$D'_2/J^2$	-0.130 768	$D'_{15}/J^3$	0.061 515	$D'_{28}/J^3$	0.584 625
$D'_3/J^2$	-0.666 667	$D'_{16}/J^3$	0.004 666	$D'_{29}/J^3$	-0.092 766
$D'_4/J^2$	-2.000 000	$D'_{17}/J^3$	-0.333 557	$D'_{30}/J^3$	-0.074 202
$D'_5/J^2$	-1.000 000	$D'_{18}/J^3$	0.042 034	$D'_{31}/J^3$	-0.194 407
$D'_6/J^3$	-0.104 778	$D'_{19}/J^3$	-2.053 736	$D'_{32}/J^3$	-2.053 736
$D'_7/J^3$	-0.032 835	$D'_{20}/J^3$	-2.053 736	$D'_{33}/J^3$	-2.053 736
$D'_8/J^3$	-0.032 835	$D'_{21}/J^3$	-1.142 275	$D'_{34}/J^3$	-1.000 000
$D'_9/J^3$	-0.519 431	$D'_{22}/J^3$	-0.396 553	$D'_{35}/J^3$	0.666 667
$D'_{10}/J^3$	-0.519 431	$D'_{23}/J^3$	-1.142 275	$D'_{36}/J^3$	0.666 667
$D'_{11}/J^3$	-0.276 601	$D'_{24}/J^3$	-0.396 553	$D'_{37}/J^3$	-2.053 736
$D'_{12}/J^3$	-0.468 697	$D'_{25}/J^3$	-0.396 553	$D'_{38}/J^3$	-0.074 202
$D'_{13}/J^3$	-0.032 279	$D'_{26}/J^3$	0.226 932	$D'_{39}/J^3$	0.000 000

$$Z^2 \Gamma_\delta^{(4)}|_{k_i=0} = \mu^{4-d} g, \quad Z \left. \frac{\partial \Gamma^{(2)}(k, \omega)}{\partial(-i\omega)} \right|_{k^2, \omega=0} = \lambda^{-1}. \quad (6)$$

This regularization of the vertex functions can be carried out in the three-loop approximation. To this end we write the values of the vertex functions in the normalization conditions as follows:

$$\begin{aligned} \Gamma_g^{(4)}|_{k_i=0} &= g_0 \sum_{i,j=0}^3 A_{ij} g_0^i \delta_0^j, \\ \Gamma_\delta^{(4)}|_{k_i=0} &= \delta_0 \sum_{i,j=0}^3 B_{ij} g_0^i \delta_0^j, \\ \left. \frac{\partial \Gamma^{(2)}}{\partial k^2} \right|_{k^2=0} &= \sum_{i,j=0}^3 C_{ij} g_0^i \delta_0^j, \\ \left. \frac{\partial \Gamma^{(2)}}{\partial(-i\omega/\lambda)} \right|_{k=0, \omega=0} &= \sum_{i,j=0}^3 D_{ij} g_0^i \delta_0^j, \end{aligned} \quad (7)$$

where the coefficients are sums of the corresponding diagrams or their derivatives at zero external momenta and frequencies. For instance, the numerical values of the derivatives of the diagrams (Fig. 1),

$$D'_i = \left. \frac{\partial D_i}{\partial(-i\omega/\lambda)} \right|_{k=0, \omega=0},$$

which comprise the coefficients  $D_{ij}$  and are obtained as a result of applying the calculation method adopted in Ref. 11, are listed in Table I, where

$$J = \int \frac{d^d q}{(q^2 + 1)^2} = \frac{S_d}{2} \Gamma\left(\frac{d}{2}\right) \Gamma\left(2 - \frac{d}{2}\right)$$

is the one-loop integral with  $S_d = 2\pi^{d/2}/(2\pi)^d \Gamma(d/2)$ ;  $\Gamma(x)$  is the gamma function. We write the expansion of  $g_0$ ,  $\delta_0$ ,  $Z$ , and  $Z_\lambda$  in powers of the renormalization coupling constants  $g$  and  $\delta$ ,

$$g_0 = g \sum_{i,j=0}^3 a_{ij} g^i \delta^j, \quad \delta_0 = \delta \sum_{i,j=0}^3 b_{ij} g^i \delta^j,$$

$$Z = \sum_{i,j=0}^3 c_{ij} g^i \delta^j, \quad Z_\lambda = \sum_{i,j=0}^3 d_{ij} g^i \delta^j, \quad (8)$$

where the unknown expansion coefficients  $a_{ij}$ ,  $b_{ij}$ ,  $c_{ij}$ , and  $d_{ij}$  can be expressed in terms of  $A_{ij}$ ,  $B_{ij}$ ,  $C_{ij}$ , and  $D_{ij}$  via the normalization conditions.

The next step in the field theoretic approach amounts to determining the scaling functions  $\beta_g(g, \delta)$ ,  $\beta_\delta(g, \delta)$ ,  $\gamma_r(g, \delta)$ ,  $\gamma_\varphi(g, \delta)$ , and  $\gamma_\lambda(g, \delta)$  that specify the renormalization group differential equation for the vertex functions:

$$\begin{aligned} \left[ \mu \frac{\partial}{\partial \mu} + \beta_g \frac{\partial}{\partial g} + \beta_\delta \frac{\partial}{\partial \delta} - \gamma_r r \frac{\partial}{\partial r} + \gamma_\lambda \lambda \frac{\partial}{\partial \lambda} - \frac{m}{2} \gamma_\varphi \right] \\ \times \Gamma^{(m)}(k, \omega; r, g, \delta, \lambda, \mu) = 0. \end{aligned}$$

For the discussion of the dynamical behavior that follows we will need only the functions  $\beta_g$  and  $\beta_\delta$  and the dynamical scaling function  $\gamma_\lambda$  determined by the following relations:

$$\begin{aligned} 4 - d + \beta_g \frac{\partial \ln Z_g}{\partial g} + \beta_\delta \frac{\partial \ln Z_g}{\partial \delta} &= 0, \\ 4 - d + \beta_g \frac{\partial \ln \delta Z_\delta}{\partial g} + \beta_\delta \frac{\partial \ln \delta Z_\delta}{\partial \delta} &= 0, \\ \gamma_\lambda &= \beta_g \frac{\partial \ln Z_\lambda}{\partial g} + \beta_\delta \frac{\partial \ln Z_\lambda}{\partial \delta}. \end{aligned} \quad (9)$$

The explicit form of the functions  $\beta_g$  and  $\beta_\delta$  in the four-loop representation was obtained by Mayer,<sup>5</sup> who introduced the coupling constants  $v$  and  $u$ , related to  $g$  and  $\delta$  by  $v = (n + 8)Jg/6$  and  $u = -16J\delta$ . Next we specify the functions  $\beta$  and  $\gamma_\lambda$ :

$$\begin{aligned} \beta_v &= v \sum_{i,j=0}^3 \beta_{ij}^{(v)} v^i u^j, \quad \beta_u = u \sum_{i,j=0}^3 \beta_{ij}^{(u)} v^i u^j, \\ \gamma_\lambda &= \sum_{i,j=0}^3 \gamma_{ij} v^i u^j; \end{aligned} \quad (10)$$

the values of the expansion coefficients for a three-dimensional Ising model ( $n=1$ ) are listed in Table II. The nature of the critical point for each value of  $n$  and  $d$  is fully

TABLE II. Values of the coefficients in the expressions for the scaling functions.

$(i,j)$	$\beta_{i,j}^{(u)}$	$\beta_{i,j}^{(v)}$	$\gamma_{i,j}$
(0,0)	-1	1	0
(1,0)	1	3/2	-0.25
(0,1)	2/3	1	0
(2,0)	-95/216	-185/216	0.053 240
(1,1)	-50/81	-104/81	0.030 862
(0,2)	-92/729	-308/729	0.008 400
(3,0)	0.389 922	0.916 667	-0.049 995
(2,1)	0.857 363	2.132 996	-0.152 964
(1,2)	0.467 388	1.478 058	-0.044 167
(0,3)	0.090 448	0.351 069	-0.012 642

specified by the stable fixed point  $(v^*, u^*)$  for the coupling constants, which is fixed by the requirement that the functions  $\beta$  vanish, i.e.,

$$\beta_v(v^*, u^*) = 0, \quad \beta_u(v^*, u^*) = 0.$$

The order of the quantities  $v^*$  and  $u^*$  is  $4-d$ , so that the expansion series in  $v$  and  $u$  for the scaling functions are asymptotically convergent if  $d=3$ .

These series are normally summed using the the Padé–Borel method.<sup>12</sup> Numerical analysis of the equations for determining the fixed points and of the stability conditions shows that in contrast to the  $\epsilon$ -expansion of Khmel’nitskiĭ<sup>2</sup> and Jayaprakash and Katz,<sup>3</sup> for  $d=3$  there is no accidental degeneracy of the fixed points at  $n=1$ . Only two of the four fixed points are of interest here: the fixed point for homogeneous systems,  $(v^* \neq 0, u^* = 0)$ , and the impurity fixed point  $(v^* \neq 0, u^* \neq 0)$ , which specifies new critical properties of disordered systems. The impurity fixed point is stable only if  $n=1$ , while for  $n \geq 2$  the presence of disorder related to the presence of frozen-in impurities is unimportant for critical behavior. The impurity fixed point for the three-dimensional Ising model in the three-loop approximation is given by  $v^* = 2.256938$  and  $u^* = -0.728168$ .

By plugging the values of the coupling constants at the fixed point into the scaling function  $\gamma_\lambda(v, u)$  we can determine the dynamical critical exponent  $z$ , which is the measure of the critical retardation of relaxation processes,

$$z = 2 + \gamma_\lambda(v^*, u^*). \tag{11}$$

However, the expansion of  $\gamma_\lambda(v^*, u^*)$  in powers of  $v^*$  and  $u^*$  at  $d=3$  is asymptotically convergent at best, and summing the series directly does not yield reasonable values. To sum the series one can employ the generalized Padé–Borel method, which amounts to applying the Borel transformation to the series

$$\gamma_\lambda(v, u) = \sum_{i,j} \gamma_{ij} v^i u^j = \int_0^\infty e^{-t} \Gamma_\lambda(vt, ut) dt, \tag{12}$$

$$\Gamma_\lambda(x, y) = \sum_{i,j} \frac{\gamma_{ij}}{(i+j)!} x^i y^j,$$

and using the Padé–Chisholm approximants

$$[M, N/K, L] = \sum_{i=0}^M \sum_{j=0}^N a_{ij} v^i u^j \left( \sum_{p=0}^K \sum_{q=0}^L b_{pq} v^p u^q \right)^{-1}.$$

The resulting expansion for  $\gamma_\lambda(v, u)$  in powers of  $v$  and  $u$  in the three-loop approximation allows using approximants of the form  $[1, 1/1, 1]$  and  $[2, 2/1, 1]$ . Application of the approximants  $[1, 1/1, 1]$  corresponds to the description of the critical dynamics of disordered magnetic materials in the two-loop approximation,<sup>13</sup> and yields a dynamical exponent  $z_{\text{imp}}^{(2)} = 2.169849$ . Using the approximants  $[2, 2/1, 1]$  makes it possible to obtain the exponent  $z$  in the form

$$z = 2 + \frac{\alpha_1 u}{\beta} + \frac{\beta - 1}{\beta^2} (\alpha_2 u^2 + \alpha_3 uv + \alpha_4 v^2) + \frac{2\beta^2 - \beta + 1}{\beta^3} (\alpha_5 u^2 v + \alpha_6 uv^2) - \frac{1}{\beta} \left[ \alpha_1 u + \frac{1}{\beta} (\alpha_2 u^2 + \alpha_3 uv + \alpha_4 v^2) + \frac{1}{\beta^2} (\alpha_5 u^2 v + \alpha_6 uv^2) \right] {}_2F_0(1, 1, \beta), \tag{13}$$

where  ${}_2F_0(1, 1, \beta)$  is the confluent hypergeometric function, and

$$\alpha_1 = \gamma_{1,0}, \quad \alpha_2 = \frac{\gamma_{2,0}}{2} - \frac{\gamma_{1,0} \gamma_{3,0}}{3 \gamma_{2,0}},$$

$$\alpha_3 = \frac{\gamma_{1,1}}{2} - \frac{\gamma_{0,1} \gamma_{0,3}}{3 \gamma_{0,2}}, \quad \alpha_4 = \frac{\gamma_{0,2}}{2},$$

$$\alpha_5 = \frac{\gamma_{2,1}}{6} - \frac{\gamma_{1,1} \gamma_{3,0}}{6 \gamma_{2,0}} - \frac{\gamma_{2,0} \gamma_{0,3}}{6 \gamma_{0,2}},$$

$$\alpha_6 = \frac{\gamma_{1,2}}{6} - \frac{\gamma_{1,1} \gamma_{0,3}}{6 \gamma_{0,2}} - \frac{\gamma_{0,2} \gamma_{3,0}}{6 \gamma_{2,0}},$$

$$\beta = \beta_1 u + \beta_2 v, \quad \beta_1 = -\frac{\gamma_{3,0}}{3 \gamma_{2,0}}, \quad \beta_2 = -\frac{\gamma_{0,3}}{3 \gamma_{0,2}}.$$

Using the values of the coupling constants at the impurity fixed point,  $v^* = 2.256938$  and  $u^* = -0.728168$ , we obtain a dynamical exponent

$$z_{\text{imp}}^{(3)} = 2.165319. \tag{14}$$

The fact that the difference in the values of  $z_{\text{imp}}$  calculated in the three- and two-loop approximations is small suggests that allowing for higher-order corrections can lead only to negligible changes in this value. At the same time, the calculations in Ref. 13 using an  $\epsilon$ -expansion in the two-loop approximation yielded  $z_{\text{imp}}^{(2)} = 2.336$ , which justifies the use of the renormalization group procedure in describing the critical behavior of dilute magnetic material for the case where  $d=3$ .

To establish the effect of impurities on dynamical critical behavior we must compare the values of  $z$  for disordered and homogeneous systems. As is known,<sup>6</sup> fluctuation corrections to the mean-field value of the dynamical exponent  $z^{(0)} = 2$  emerge in homogeneous systems only in the two-loop approximation, while in disordered systems the dynamical

TABLE III. Values of the coefficients in the expressions for the vertex functions.

Coefficient	$d=3$	$d=2$
$A_1$	-1.0	-1.0
$A_2$	1.222 2222	1.375 0699
$A_3$	-1.705 3479	-2.305 4548
$B_1$	0.005 4869	0.008 4916
$B_2$	-0.007 0112	-0.011 6591
$B_3$	0.010 1430	0.017 9966
$C_1$	0.009 6865	0.015 2547
$C_2$	-0.012 6257	-0.021 3740
$C_3$	0.016 9420	0.035 2450

cal effects of scattering of magnetization fluctuations by impurities show up by the first-order approximation.

The effectiveness of summation methods for asymptotically convergent series is largely determined by the number of known terms in the series. Therefore, when the summation methods are applied to the dynamical scaling function, the accuracy of the three-loop approximation for the exponent  $z$  of a disordered system can only correspond to the four-loop approximation for  $z$  in a homogeneous spin system. With this mind, we calculated the dynamical critical exponent for homogeneous three-dimensional spin systems in the four-loop approximation. The expressions for the vertex functions in (7) for homogeneous systems become much simpler, and in the four-loop approximation assume the form

$$\Gamma_v^{(4)}|_{k_i=0} = v_0 + A_1 v_0^2 + A_2 v_0^3 + A_3 v_0^4,$$

$$\left. \frac{\partial \Gamma^{(2)}}{\partial k^2} \right|_{k^2=0} = 1 + B_1 v_0^2 + B_2 v_0^3 + B_3 v_0^4,$$

$$\left. \frac{\partial \Gamma^{(2)}}{\partial(-i\omega)} \right|_{k=0, \omega=0} = 1 + C_1 v_0^2 + C_2 v_0^3 + C_3 v_0^4, \quad (15)$$

where  $v_0 = (n+8)Jg_0/6$ . The values of the coefficients at  $n=1$  are listed in Table III. The four-loop diagrams that produce the coefficient  $C_3$  are shown in Fig. 2. Carrying out the calculations, these diagrams split into 48  $4d$ -fold integrals, whose numerical values are listed in Table IV. To describe the dynamical behavior we require only the functions  $\beta(v)$  and  $\gamma_\lambda(v)$ :

$$\beta(v) = -(4-d) \left[ \frac{\partial \ln Z_v v}{\partial v} \right]^{-1}, \quad \gamma_\lambda(v) = \beta(v) \frac{\partial \ln Z_\lambda}{\partial v}. \quad (16)$$

The explicit form of the first function in the six-loop approximation was obtained by Baker *et al.*<sup>12</sup> By consistently applying the above field theoretic approach, we were able to derive an expression for the dynamical scaling function  $\gamma_\lambda(v)$  in the four-loop approximation:

$$\begin{aligned} \gamma_\lambda(v) = & -(4-d)v[2(B_1 - C_1) + (3B_2 - 3C_2 - 4A_1B_1 \\ & + 4A_1C_1)v + (4B_3 - 4C_3 - 9A_1B_2 + 9A_1C_2 \\ & + 10A_1^2B_1 - 10A_1^2C_1 - 4A_2B_1 + 4A_2C_1 \\ & - 8B_1D_1 + 6B_1^2 - 2C_1^2)v^2]. \end{aligned} \quad (17)$$

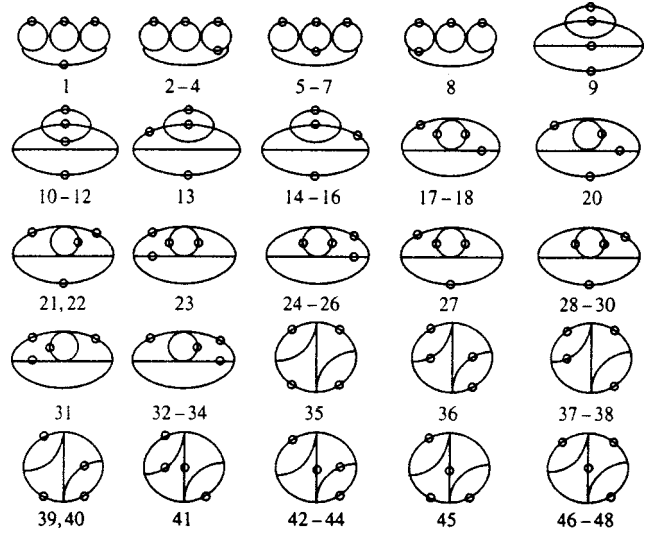


FIG. 2. Four-loop diagrams contributing to the vertex function  $\Gamma^{(2)}(k, \omega)$ .

Inserting the values of the coefficients listed in Table III for  $d=3$  and  $n=1$ , we obtain

$$\gamma_\lambda(v) = 0.008399v^2 - 0.000045v^3 + 0.020423v^4. \quad (18)$$

In accordance with Ref. 12,

$$\beta(v) = -v + v^2 - 0.422497v^3 + 0.351069v^4 - 0.376527v^5. \quad (19)$$

To calculate the values of the coupling constant  $v^*$  at the fixed point and the dynamical critical exponent  $z$ , we used the Padé-Borel summation method with the approximants  $[4/1]$  and  $[3/1]$ , respectively. As a result, for  $d=3$  and  $n=1$  we obtained

$$v^* = 1.4299, \quad z_{\text{pure}}^{(4)} = 2.017.$$

Comparison of the results revealed a significant difference between the values of the dynamical exponent  $z$  for the homogeneous and disordered Ising models. This makes it possible to study the effect of impurities on the dynamical critical behavior in a real physical experiment and via Monte Carlo simulation.

Let us compare our value of the dynamical exponent  $z_{\text{imp}}^{(3)}$  with computer simulations of the dynamical critical behavior of a disordered Ising model.<sup>14-16</sup> In Refs. 14 and 15, critical magnetization relaxation was numerically simulated for a system with dimensions of  $48^3$  and impurity concentrations  $0.4 \leq p \leq 1$ . Jan *et al.*<sup>17</sup> combined the Monte Carlo method with the dynamical renormalization group method to determine the dynamical critical exponent  $z$ . The following values of the critical exponent were obtained for the homogeneous system and two slightly disordered systems with  $p = 0.95$  and  $0.8$ :

$$\begin{aligned} z(1.0) &= 1.97 \pm 0.08, \quad z(0.95) = 2.19 \pm 0.07, \\ z(0.8) &= 2.20 \pm 0.08, \end{aligned}$$

which are in good agreement with the numerical results. Heuer<sup>16</sup> obtained the values of  $z$  by analyzing the asymptotic properties of the dynamical autocorrelation function for a

TABLE IV. Values of four-loop diagrams.

No.	$d=3$	$d=2$	No.	$d=3$	$d=2$	No.	$d=3$	$d=2$
1	0.104 869	0.165 307	17	0.001 108	0.004 131	33	0.002 527	0.007 463
2	0.004 166	0.009 670	18	0.000 923	0.003 307	34	0.014 580	0.029 449
3	0.008 180	0.022 921	19	0.000 932	0.003 343	35	0.039 776	0.070 254
4	0.029 674	0.059 714	20	0.019 410	0.034 609	36	0.002 378	0.006 421
5	0.003 264	0.003 943	21	0.019 189	0.034 135	37	0.004 691	0.012 723
6	0.015 354	0.010 076	22	0.004 177	0.011 294	38	0.003 820	0.007 370
7	0.014 330	0.028 777	23	0.001 928	0.004 644	39	0.011 650	0.027 311
8	0.011 627	0.016 314	24	0.000 706	0.005 891	40	0.005 377	0.013 297
9	-0.002 506	-0.006 853	25	0.003 421	0.010 167	41	0.003 981	0.007 464
10	0.000 823	0.002 744	26	0.000 862	0.003 535	42	0.003 314	0.010 303
11	0.003 444	0.009 238	27	0.000 551	0.002 471	43	0.009 470	0.023 519
12	0.003 745	0.010 685	28	0.003 898	0.011 209	44	0.003 866	0.010 905
13	-0.004 883	-0.012 280	29	0.001 077	0.003 405	45	0.023 730	0.038 420
14	-0.004 883	-0.012 280	30	0.003 815	0.011 007	46	0.033 485	0.062 921
15	0.007 527	0.017 1805	31	0.007 379	0.012 666	47	0.007 121	0.021 633
16	-0.005 471	-0.014 199	32	0.004 177	0.009 667	48	0.004 760	0.011 691

system that is in a state of equilibrium and exhibits strong magnetization fluctuations. For instance, it was found that

$$z(1.0) = 2.095 \pm 0.008$$

for the homogeneous system

$$z(0.95) = 2.16 \pm 0.01, \quad z(0.9) = 2.232 \pm 0.004,$$

$$z(0.8) = 2.38 \pm 0.01,$$

for slightly disordered systems, and

$$z(0.6) = 2.93 \pm 0.03$$

for a system with  $p=0.6$ . Believing that the fixed point of the critical behavior of a slightly disordered system, which is independent of the impurity concentration, is also such a point for any impurity concentration, Heuer<sup>16</sup> estimated the asymptotic value of the dynamical exponent  $z$  to be  $2.4 \pm 0.1$ . The value of  $z$  for a homogeneous system obtained by Heuer<sup>16</sup> differs drastically from the results of the field theoretic approach, while for a system with  $p=0.95$  the agreement is unexpectedly good. Our view on the universality of the critical behavior of disordered systems has been explained in Refs. 14 and 15, where we proposed separating the universal critical behavior of slightly disordered systems from that of highly disordered systems and hypothesized that the critical exponents of three-dimensional disordered systems exhibit stepped universality.

The predictions of the theory concerning the effect of impurities on the dynamical critical behavior of magnetic materials (a higher value of  $z_{\text{imp}}(d=3)$  compared to the value of  $z_{\text{pure}}(d=3)$ ) can be corroborated by several experimental methods: inelastic neutron scattering (the linewidth  $\omega_\varphi \propto |T - T_c|^{z\nu}$  at  $q=0$  and  $\omega_\varphi \propto q^z$  at  $T=T_c$ ), EPR and NMR (the resonance linewidth  $\Delta\omega \propto |T - T_c|^{(d-2+\eta-z)\nu}$ , where  $\eta$  is the Fisher exponent), measurements of the dynamic susceptibility in an external high-frequency magnetic field ( $\chi(\omega) \propto \omega^{-\gamma/z\nu}$  at  $T=T_c$ , where  $\gamma$  is the susceptibility exponent), and ultrasound measurements (the sound absorption coefficient  $\alpha(\omega) \propto |T - T_c|^{-(\alpha+z\nu)} \omega^2 g(\omega/|T - T_c|^{z\nu})$  and the acoustic dispersion  $C^2(\omega - C^2(0)) \propto |T - T_c|^{-\alpha f(\omega/|T - T_c|^{z\nu})}$ ). Unfortunately, we know of no ex-

perimental work in which the dynamical critical behavior of slightly diluted Ising-like magnetic materials is studied.

The critical dynamics of a slightly disordered two-dimensional Ising model in the relaxation regime does not differ from the dynamics of the homogeneous model.<sup>13</sup> An analysis of the critical dynamics of the two-dimensional Ising models shows that the values  $z$  span a broad range:  $2.08 \leq z \leq 2.24$ . For instance,  $z = 2.14 \pm 0.02$  (Ref. 18),  $2.13 \pm 0.03$  (Ref. 19),  $2.076 \pm 0.005$  (Ref. 20),  $2.24 \pm 0.04$  (Ref. 21),  $2.24 \pm 0.07$  (Ref. 22), and  $2.16 \pm 0.04$  (Ref. 23) in computer simulation;  $z = 2.126$  (Ref. 24) in the field theoretic approach in the two-loop approximation with the interpolation of the results of the  $1 + \varepsilon$ - and  $4 - \varepsilon$ -expansions; and  $z = 2.183 \pm 0.005$  (Ref. 25) in the same approach with interpolation of the results of the high-temperature expansion.

We calculated the dynamical exponent  $z$  for a homogeneous two-dimensional Ising model in the four-loop approximation in the context of the field theoretic approach. The corresponding values of the coefficients in the expressions (15) for the vertex functions and the numerical values of the four-loop diagrams for the two-dimensional model are listed in Tables III and IV. As a result we arrived at the following expressions for the scaling functions:

$$\gamma_\lambda(v) = 0.027053v^2 - 0.004184v^3 + 0.022130v^4,$$

$$\beta(v) = -v + v^2 - 0.716174v^3 + 0.930766v^4 - 1.582388v^5. \quad (20)$$

Summing by the Padé-Borel method, we found the values of the coupling constant  $v^*$  at the fixed point and of  $z$ :

$$v^* = 1.8836, \quad z^{(4)}(d=2) = 2.093.$$

We see that the exponent  $z$  is at the lower edge of the range mentioned earlier. The adopted procedure of calculating the exponents is assumed to be the most accurate, so that we expect the calculated values to be the benchmarks for computer simulations of homogeneous systems and to be used in developing simulation methods for disordered systems.

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